Mathematics

Grand Valley State University
Fred Meijer Center for Writing and Michigan Authors

The Mathematical Proof

An Overview of the Mathematical Proof

The mathematical proof is a common assignment across a number of mathematics courses beginning in MTH 210, Communicating in Mathematics. A mathematical proof indicates that you have a thorough understanding of the conjecture in question by communicating the process by which you prove or disprove its veracity. The purpose of writing a proof is to demonstrate that you've gained a deeper knowledge and understanding of mathematical concepts and perhaps have made connections to other field of mathematics by communicating insights to the larger mathematics community.

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Audience

As you write the proof, keep in mind that your primary audience consists of your professor and fellow classmates – an academic audience. You should present detailed information and communicate results clearly so that your colleagues can follow your mathematical course of action.

Writing Process Tips

Consider your first draft as a "write up" of the process you followed in determining whether a given conjecture is true or false and the subsequent steps taken in writing a proposition or counterexample. Remember to refer to the guidelines in your textbook for each proposition, theorem, or counterexample you write.

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Proofread and Revise

As you proofread and revise your first draft, keep in mind that your audiece should be able to understand the reasoning in the proof. (Be clear and concise in your wording. Include only details that are necessary for your audience to understand your approach to proving the proposition.) Use full declarative sentences and avoid overly long wordy phrases.

Proofread, revise, and edit. Make sure you use commas appropriately and double check for misspelled words rather than relying on the grammar and spell check. Don't rush through your draft during this process. Remember your goal is to effectively communicate a mathematical process that others may or may not be familiar with so good grammar is especially important. Keep paragraphs short and skip lines between each paragraph to ensure that your audience can follow the sequence of your proof.

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One effective proofreading strategy is to read your work aloud including the meaning of all mathematical symbols, equations, and expressions.

Appropriate Word Processor

Your proofs must be written with a word processor capable of producing the appropriate mathematical symbols and equations such as Microsoft Word's Equation Editor or LaTeX. Additional detail regarding symbols can be found in the Equation Format section.

Voice & Tone

Write in a formal academic voice. This does not mean you should pack the writing with jargon or complicated sentences; rather, focus on accuracy, clarity, and detail.

Write in the present tense/active voice in your construction of the proof. Use the prounoun "we" generously to encourage your audience to participate in the discussion. For help identifying active and passive voice see our handout at http://www.gvsu.edu/wc.

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Components of Mathematical Proofs

Proposition

Think of the first lines of your proof as an introduction opening up a discussion written in a specific format. Your proposition should be constructed as a declarative statement in your own words rather than repeating a claim found in a textbook or handout. A carefully worded direct statement of at most two or three sentences will best convey the proposition at hand.

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Examples

Proposition 1. If x and y are odd integers, then $x \cdot y$ is an odd integer.

Proposition 2. For a right triangle, suppose that the hypotenuse has a length of c feet and the lengths of the sides are a feet and b feet. If the triangle is an isosceles triangle, then the area of the right triangle is $\frac{1}{4}c^2$.

Proof

Write the proof in the same direct and declarative manner as your proposition. Begin this section with a statement of what is being assumed. A good way to do this is to write "We assume that..." Follow this with a sentence of what you intend to prove using a phrase such as "We will prove that..."

Write the proof in the same direct and declarative manner as your proposition. Begin this section with a statement of what is being assumed.

Examples

Proof. We assume that x and y are odd integers and will prove that $x \cdot y$ is an odd integer.

Proof. We assume that the hypotenuse of a right triangle has a length of c feet, the legs have lengths a feet and b feet, and the triangle is isosceles. We will prove that the area of the right triangle is $\frac{1}{4}c^2$.

At the beginning of a proof, you should make sure it is clear what proof method you intend to use. For example, if you are proving the contrapositive, using a proof by contradiction, or using a proof by mathematical induction, be sure to signify your intentions. Try sentences like the following:

- We will prove this result by proving the contrapositive of the statement.

- We will prove this statement using a proof by contradiction.
- We will assume to the contrary that...
- We will use mathematical induction to prove this result.

Proof Sequence

As you work through your proof, ensure that your readers know the status of each assertion you make. Is your assertion an assumption of the theorem or proposition, a previously proven result, a well-known result, or something from the reader's mathematical background? Your audience requires explicit and meticulous details for each step you take in writing your proof. Keep in mind the readers mathematical background.

Example

The area of a triangle is one half the base times the height. For a right triangle, the base and height are the length of the legs. So the area of the right triangle is $area = \frac{1}{2}ab$.

Refer explicitly to definitions when appropriate so that your audience has a clear understanding of your course of action.

Example

The definition of an isosceles triangle tells us that the legs have equal lengths. Since , $a = b$, we can rewrite the area equation as follows:

$$area = \frac{1}{2}a^2$$

If you use the previous results or a theorem as a justification for a step in your proof, you should reference the theorem and explain how the hypotheses are satisfied before drawing your conclusion. Also, take note of the format:

Example

The Pythagorean Theorem tells us that $a^2 + b^2 = c^2$.

The fact that this is an isosceles triangle allows us to conclude that $a = b$.. We can then make a substitution in the previous equation from the Pythagorean Theorem to give us $2a^2 = c^2$, or in other words

$$a^2 = \frac{1}{2}c^2.$$

Using substitution, we then get

$$area = \frac{1}{2}\left(\frac{1}{2}c^2\right).$$

Writing Tips

- Ensure that your readers know the status of each assertion you make.
- Include explicit and meticulous details for each step you take in writing your proof.
- Take note of the format.

Conclusion

After completing your proof, you should clarify for your reader that you have finished. You may use the "end of proof symbol" ■ and write "this completes the proof," or state in a sentence what you have proven or *Q.E.D.* Doing so keeps your audience involved in the mathematical process while confirming that you're finished working through your proof.

Example

We have just proven that if a right triangle is an isosceles triangle, then the area of the right triangle is $\frac{1}{4}c^2$, where *c* is the length of the hypotenuse in feet. ■

You might consider double checking your proof by working backward from your conclusion. It is often easier to see the very same paths if you work in the opposite direction from what you've just proven.

At this point, you might consider double checking your proof by working backward from your conclusion. It is often easier to see the very same paths if you work in the opposite direction from what you've just proven.

Equation Format & Technical Mathematical Tips

• In Microsoft Word, the Equation Editor is the easiest way to format mathematical symbols and equations. Otherwise, LaTeX is a common proof writing tool. If you're using a word processor, be sure that all equations correspond to mathematical typeset so that variables are italicized, vectors are in boldface, and mathematical terms are in regular font. For example, be sure to write sin (*x*) and not sin (x) or *sin* (x).

Be sure that all equation correspond to mathematical typeset.

• Important equations and manipulations should be centered and double spaced before and after each equation or manipulation. If the left side of an equation does not change, it does not need to be repeated.

Example

Through algebra, we obtain

$$\begin{aligned} x \cdot y &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1 \end{aligned} .$$

Since *m* and *n* are integers, we conclude that . . .

• Use an equation number if you refer to a previous equation later in a proof. Number only those equations you refer to later in the proof and do not capitalize the word "equation" when referring to is by number.

Format the equation as you normally would but with the equation number written in parentheses on the same line in the right hand margin.

Example

Since *x* is an odd integer, there exists an integer n such that

$$x = 2n + 1 . \qquad (1)$$

Later in the proof, there may be a line such as:

Then, using the result in equation (1), we obtain . . .

• When using quantifiers, always use appropriate words for the quantifiers. Use "$x = 2n + 1$, for some integer n" in order to point out the existential quantifier with the word "some." Also, use a phrase such as "for each integer n" (or "there exists an integer") when using a unviersal or existential quantifier. Respectively, consider the statement, "For an integer m, $m^2 + m$ is even." It is not clear whether an existential quantifier or a universal quantifier is implied by the phrase, "for an integer m."

Avoid the word "where" as it usually means the writer has failed to identify a quantifier. Be careful when using the word "a" or "an" in statements that should have a quantifier.

• In formal mathematical writing use English to convey your meaning instead of special symbols. It is often easier to read and write the English words instead of cumbersome symbols:

Use:

For each real number x, there exists a real number y such that $x + y = 0$.

Instead of: $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = 0)$

• Do not use the * symbol for multiplication, the ^ symbol for exponents, or the / symbol for division when using a complex fraction.

• Do not use a mathematical symbol at the beginning of a sentence as it appears grammatically awkward.

Prompts for Writing Consultations

• Is there a logical pattern to working through the proof? Is each assertion justified and logically connected to the next step?
• Are equations fully interpreted and understandable?
• Do mathematical symbols, notations, and equations make sense when read aloud in the context of surrounding sentences?
• Is the writing free of jargon and complicated sentences?
• Is the writing clear and well edited? Are paragraphs short? Double check for appropriate use of commas and misspelled words.
• Is sentence structure standard and complete?
• Are the theorem and proof clearly stated declarative sentences written in the present tense/active voice?
• Does the writer use "we" throughout instead of "I?"
• Does the proof sentense look similar to: "We assume that... and we will prove that ...?"
• Is each proposition numbered and each proof labeled?
• Are equations centered with spaces separating assertions?
• Did the writer use Microsoft Word Editor or LaTeX to format equations? If not, make certain all equations are formatted with the appropriate mathematical symbols and equations.
• Does the proof end with the "end of proof symbol" ■ and state "This ends the proof?"