January 5, 2006

Provost Gayle Davis
Grand Valley State University
Allendale, MI 49401

Dear Provost Davis:

I would like to describe for you my activities during my sabbatical leave of the Fall semester of 2006. This leave was intense, productive and very fruitful and I thank you and the University for giving me this valuable opportunity. I spent the entire fall term at Cornell University in Ithaca, New York working towards two goals.

The main goal of the sabbatical was to create a dynamic, internet accessible[1] and user friendly Java program and applet to explore, manipulate, and visualize equal (two dimensional) circle packings in various domains. I have achived this goal and have written approximatly 21,600 lines of Java code spread over 57 classes to create the program called EZPack. The main features of EZPack are the following:

- The user may explore packings of equal circles in any parallelogram with a set of opposite sides identified, opposite sides identified with a twist or any subset of the sides self-identified. This results in 36 different packing domain types and was a significant issue to handle programatically. By explore I mean that the user can drag any circle around the domain and the program will update the radius and the display of the circle (i.e. it takes into account the identifications on the boundary, if any).

- The user can set the attributes of any circle to create a display that is completely customizable and instructive for the packing under study. Further, these display choices may be printed to postscript (a printer language used by many printers and editing programs).

- The user can load and save any packing created with the program. Also the user can import packings that are specified in text files that contain the locations of the centers of circles in the packing.

[1] This program is availible online at the address `http:\\faculty.gvsu.edu\dickinsw\EZPack`.

- The user can adjust the number of decimal places that the program uses to locate the centers of the circles and to calculate the common radius of the circles. This was another challenge to deal with in writing the program because the native code of Java supports a precision of about 13 decimals places and I had to write code to handle an arbitrary number of decimal places. (In practice, keeping about 50 decimal places is all that is required.)

- The program computes several useful graphs associated with a given packing: the Delaunay triangulation, the Voronoi diagram and the packing graph.

The other feature of the program is that it is written to be extensible. That is, there are two sets of classes that can be super-classed in the future to extend the functionality of the program. One set of classes will enable the program to model packings in non-parallelogram domains and another set will enable a user (possibly a student) to implement a new packing algorithm.

The secondary goal of the sabbatical was to develop and collect questions suitable for undergraduate research. To this end I worked very productivly with Dr. Robert Connelly (of Cornell University) during the last third of leave. During this time we met almost daily, studied the problems surrounding packings of equal circles on a equilateral triangular torus and developed a plan that might possibly prove the Toth conjecture. In addition to these meetings and thinking about the problems we discussed, to help me understand more about the tools that we were using to to analyze this area of mathematics I had to read a number of mathematical papers. The following details the papers I read.

- The tensegrity approach to equal circle packings is outlined in [4], [3] and [1]. This was not familar to me and I had to read deeply into each of these papers.

- In order to understand some of the special arrangements (counterexamples to the original crystalization conjecture) and to independently verify them I had to read [6] in extreme detail. This is written for the granular matter audience, making more challenging to read than usual. I disagree with many of the mathematical assertions with regard to their discovery of periodically stable arrangments. Independent verifica-

tion rested heavily on several Maple worksheets (a program for doing and checking mathematics symbolicaly) that I created and a careful reading of [7], and [2].

- The subject of tensegrity frameworks in general is covered in [5], and I had to read the first part of this paper to understand the idea of stresses and some of the notation of [6].

- Professor Connelly has given numerous lectures related to this area of mathematical research and was kind eough to share his lecture notes with me. I spent many hours closely reading these notes.

Additionally, I spent time searching the literature for (and obtaining) manuscripts related to different approaches to the packing of equal circles.

This sabbatical has allowed me to create a Java program that will be the backbone tool for my research with undergraduates and to formulate many questions (and equally important, possible approaches to those questions) for undergraduate research. These are activities that would not have occured without a sabbitical, will enable me to prusue a new direction of undergraduate research for about the next five years and will enhance the quality of education of many mathematics students at Grand Valley.

Sincerely,

William Dickinson
Department of Mathematics

cc: President Haas
Dean Antczak, College of Liberal Arts and Sciences
Research and Development Center
Library Archives

References

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