Sabbatical Leave Request
for Winter Semester 2000

Dr. Theodore A. Sundstrom
Department of Mathematics and Statistics

1. **Descriptive Title of the Project**

   Mathematical Reasoning and Communication: Exploration, Proof, and Writing.

2. **Goals and Objectives**

   The focus of this sabbatical proposal is the course, MTH 210 - Communicating in Mathematics. This is a very important course in the mathematics curriculum at Grand Valley State University. It is a required of all mathematics majors and is a prerequisite for many of the upper level mathematics courses. Its importance, however, is not only in the mathematical content in the course. It is important because it is the first course where the focus is on the formal development of mathematics. It is the course that the department uses to teach students how to be able to read and understand mathematical definitions and proofs and how to write their results according to accepted guidelines so that their work (and reasoning) can be understood by others. This course sets the expectations for the upper level mathematics courses.

   This project has three primary goals. They are:

   1. To develop a document to be approved by the Department of Mathematics and Statistics that will describe:

      - The objectives of MTH 210 - Communicating in Mathematics.
      - The writing requirements for this course.
      - How the writing requirements satisfy the SWS requirements as established by Grand Valley State University.
      - The types of proof techniques that will be studied in this course.
      - The mathematical content that will be studied in this course.

      This document will be for the faculty of the Department of Mathematics and Statistics who will teach this course and the upper level courses for which MTH 210 is a prerequisite.

2. To develop a second document, similar to the first, but intended for mathematics majors and minors who take MTH 210. In addition, this document will be useful to them in their upper level mathematics courses.

   In addition, this document would be made available to the Writing Center. It is hoped that this document would be useful to the tutors in the Writing Center as they assist students in MTH 210.

3. To develop a complete set of instructional materials for this course that will incorporate the expectations for the course that will be in the document described in the first goal.

I do not plan to complete this project by myself. It is intended that work on the first two goals will be a collaborative effort with the faculty of the Department of Mathematics and Statistics. (See Project Plan and Timetable below.) More importantly, completion of all three goals will be a collaborative effort with Dr. Karen Novotny. (A letter of support from Dr. Novotny is attached at the end of this proposal.) Over the past ten years, Dr. Novotny and I have taught MTH 210, Communicating in Mathematics, more than anyone else in the department. Although we teach the course in somewhat different manners, I think it is fair to say that Dr. Novotny and I have similar goals and expectations for this course.

With the recent increase in the number of sections of MTH 210 taught each year and with the recent increase in the number of faculty in the Department of Mathematics and Statistics, more and more faculty are being assigned to teach this course. Many of these faculty have been hired in the past few years and were not here when the course was being developed or when the course was significant revised in 1994. (Dr. Virginia Muraski, Professor Emeritus, was instrumental in developing this course in the late 1970's, and Dr. Novotny and I were deeply involved in the revision of the course in 1994.) Consequently, there is a real need to re-establish writing guidelines and content guidelines for this course.

The writing guidelines are very important because MTH 210 is a Supplemental Writing Skills (SWS) course. The faculty of the Department of Mathematics and Statistics have decided to have this SWS course in the mathematics major at the lower level rather than the upper level to emphasize the importance of writing in mathematics. If students are taught good (mathematical) writing habits early, we can expect better writing from students in our upper level courses. We can also reinforce the writing skills learned in MTH 210. The goal

is that when students complete the mathematics major, they will have the ability to effectively write mathematical exposition.

From an institutional point of view, the first two goals of this project will help support recommendations concerning student evaluation standards as stated on page 15 in the Institutional Plan for Grand Valley State University dated June 1997. The recommendations these goals support are:

4.2.2 Each academic unit will develop explicit standards for student academic achievement in the courses of their disciplines and for each major. Guidelines for these standards will be developed by the appropriate division and school in conjunction with Academic Governance. Units will develop plans to inform students of these standards.

4.2.3 Each academic unit will also review how current standards for courses that are also part of University wide programs (such as General Education, Basic Skills, and the Honors Program) are communicated to students.

Although not explicitly stated in Objective 4.2.3, this objective also applies to the Supplemental Writing Skills Program at Grand Valley.

From a personal, instructional point of view, the ultimate goal of this project is the third goal: to develop a complete set of instructional materials for MTH 210. Although we are not yet certain what form these materials will take, Dr. Novotny and I have agreed that our goals for the course materials that we write for MTH 210 are as follows:

- We will have complete instructional materials for ourselves when we teach the course. These materials will be written so that they are consistent with the guidelines that will be approved by the Department of Mathematics and Statistics.

- These materials will be available to any faculty member teaching MTH 210 at Grand Valley State University. If possible, we would like to have an “electronic” version available to all faculty members. In this way, individual faculty members could use selected portions of the materials and could even modify them to suit their own instructional methods.

Once these materials are completed (or nearly completed), Dr. Novotny and I may consider contacting commercial publishers to determine if there is any interest in publishing these materials.

This project will satisfy several of the conditions and criteria for a sabbatical leave as stated in the Administrative Manual (Chapter 4, Section 2.3.4.D). It is a synthesis or development of prior efforts or experiences since I have taught MTH 210 several times and have never found existing textbooks adequate. (This will be described in detail in Part 5, Evidence of Academic Preparation.) The project is also concerned with a significant problem in the mathematical education and preparation of mathematics majors and in completing the project, I will be using skills that will deepen and extend my teaching capabilities. Finally, there is a possibility of a significant contribution to the methods used to teach MTH 210 at Grand Valley State University.

3. **Project Plan and**
4. **Timetable**

Phase 1 - Fall Semester 1998

During this phase of the project, Dr. Novotny and I plan to engage in several discussions about this course. We will compare our approaches to the course, our expectations of students in the course, our teaching techniques, and the mathematical content we have used in the course. Our discussions will focus on the goals of the project as described in Part 2 of this proposal. At the end of this phase, we should have a set of materials that includes (most) everything that each of has used in the course during the past several years.

Phase 2 - Winter Semester 1999

Once Phase 1 is completed, Dr. Novotny and I will initiate discussions about MTH 210 with other interested faculty in the Department of Mathematics and Statistics. The plan is to have preliminary consensus regarding the first goal of the project. That is, we hope to have consensus regarding:

- The objectives of MTH 210 - Communicating in Mathematics.
- The writing requirements for this course.
- How the writing requirements satisfy the SWS requirements as established by Grand Valley State University.
- The types of proof techniques that will be studied in this course.
- The mathematical content that will be studied in this course.

This phase of the project is crucial since we do not want to begin developing instructional materials for the course until there is some departmental consensus regarding these items.

Phase 3 - Summer 1999 and Fall Semester 1999

During this phase, Dr. Novotny and I plan to write the two documents described in the first two goals of the project. The goal here would be to bring these documents to the department for approval during the Fall Semester of 1999.

These documents will provide the guidelines we need to design and develop the instructional materials for the course. At this time, we would also have our own materials that we have used for MTH 210. With the approved guidelines for the course, we will be able to decide which of these materials we could use. I would anticipate that we will only have a small portion of the materials for the course and those that we do have will have to be rewritten. The goal for this phase is to decide:

- In what format the instructional materials should be written.
- What materials that we currently have could be rewritten in this format.
- What remains to be written.

Also, during the summer of 1999, I plan to write a few preliminary versions of course materials that I could use in MTH 210. I plan to request teaching a section of MTH 210 during the Fall Semester of 1999 and to test the materials that I have written.

Phase 4 - Winter Semester 2000

This will be the actual time of the Sabbatical Leave. During this time, it will be my responsibility to write first drafts of the materials for the course according to the guidelines approved by the Department of Mathematics and Statistics and according to the plan developed by Dr. Novotny and myself for the design and development of these materials.

Dr. Novotny would review, edit, and suggest ways to improve these materials. With her feedback, I would then complete the first version of the materials.

The plan is to have a complete set of materials to use for MTH 210 by the beginning of the Fall Semester of 2000. Again, I would request that the department assign me to a section of MTH 210 for that semester so that I could class-test the materials.

5. Evidence of Academic Preparation

During the past ten years, I have been deeply involved in teaching MTH 210 - Communicating in Mathematics. In addition, I was one of the faculty members most involved in developing the revisions to the course that were approved in 1994.

As has been stated before, MTH 210 is an extremely important course in the curriculum for a mathematics major (or minor) at Grand Valley State University. It is the course that serves as a transition from the problem oriented calculus courses to the more theoretical oriented upper level courses. This can be a difficult transition for calculus students since the primary objectives of calculus courses are: (1) the development of students' understanding of the concepts of calculus from a geometric and numerical viewpoint; and (2) improving students' problem solving skills. As a result of this, most mathematics students do not encounter any formal proofs in their first year of calculus. However, in many upper level mathematics courses, the focus is more on the formal development of mathematical structures and theory. Students are expected to be able to read and understand proofs, construct their own proofs, and write their results in a way that other people can understand their reasoning.

When I have taught this course, I have included the following objectives in my syllabus:

1. To improve the quality of communication in mathematics. This includes improving writing techniques, reading comprehension, and oral communication in mathematics.
2. To better understand the true nature of mathematics and its language.
3. To provide opportunities for the development of talents for creative thinking, problem solving, and writing proofs.
4. To explore and understand the concepts listed in the tentative course content below.

(See Appendix C, MTH 210 Syllabus, Fall 1996.) In addition, every section of this course taught at Grand Valley is an SWS (Supplemental Writing Skills) Course and must satisfy all the requirements of being an SWS course.

The importance of this course can also be seen by examining the "Desired Outcomes for Mathematics Majors" as stated in the Department of Mathematics and Statistics Assessment Plan. These desired outcomes are:

- Mathematics graduates will be able to communicate mathematics effectively both orally and in writing.
- Mathematics graduates will have been provided with experiences that actively engage them in doing mathematics and that enable them to: explore and analyze mathematical situations; collect, present, and interpret data; make conjectures; reason logically, present arguments and construct mathematical proofs; and use a variety of mathematical methods to solve problems.
- Mathematics graduates will have the foundation and desire to continue their study and to expand their expertise both with mathematics and the use of mathematics in other fields.

These desired outcomes for mathematics graduates can almost be the course objectives for MTH 210. In fact, MTH 210 lays the groundwork for the first two of these desired outcomes.

The objectives for MTH 210 make it quite difficult to find a suitable text for this course. In discussions that I have already had with Dr. Karen Novotny, we have identified the following problems with the textbooks that have been used for the course: (A list of some textbooks that have been used for the course is given in Appendix A, Bibliography.)

1. The course we teach is an SWS course. Most textbooks do not really address the issue of writing mathematical exposition and so they do not address the first objective of the course. This is, of course, something that can be handled by the instructor during the course and all of us who have taught the course have attempted to do this. However, it would be nice if the course materials addressed this specifically and then used practices of good writing throughout the textbook. Early in the course, I distribute a paper whose title is "Guidelines for Writing Mathematical Exposition." (See Appendix D.) I make every attempt to follow these guidelines in any writing that I do for the course and distribute to students. However, the writing in most textbooks does not follow these guidelines.

2. One of the primary objectives of MTH 210 is to develop student's ability to construct mathematical proofs and then to write the proof in a coherent manner that conveys an understanding of the proof to the reader. These are two distinct skills and we attempt to address both in MTH 210. Unfortunately, most textbooks do not address the actual process of writing a proof. There are textbooks that do discuss how to construct a proof but for the most part, they do not provide any guidelines for writing the proof.

3. Those textbooks that deal with issues related to constructing proofs usually do not cover material that the department would like to have included in this course. Last spring, we had a discussion about this course in a group that included those who have taught MTH 210 and those who teach courses for which MTH 210 is a prerequisite. The result was that MTH 210 should include the following:

   List #1 - Methods of Proof
   - Direct proof.
   - Proof by contrapositive.
   - Proof by contradiction.
   - Proof using cases.
   - Proof by mathematical induction.
   - Constructive proofs versus existence proofs.

   List #2 - Content
   - Logic.
   - Quantifiers with an emphasis on negations.
   - Elementary set theory.
   - Functions (including one to one, onto, inverse functions).
   - Number theory including congruence arithmetic.

   List #3 - Optional Content
   - Relations and properties of relations.
   - Equivalence relations.
   - Cardinality.

   The core content for the course would be that in List #2. This is not a large amount of content for the course since the course must also deal with the items in List #1.

4. The methods used to teach mathematics courses have been changing. A major emphasis now is to have the students explore mathematical concepts rather than having the text or the instructor tell them about these concepts. The attempt is to have the students actively involved in the learning process. This often involves collaborative learning where

students work in groups to brainstorm, make conjectures, test each others' ideas, reach consensus, and hopefully, to develop sound mathematical arguments to support their work. Many textbooks for this "transition" course are quite traditional and simply present the material and then have the students attempt to work exercises or write proofs. Some newer textbooks do have an emphasis on exploration and conjecture in mathematics. However, these textbooks often do not have the mathematical content that we want for this course. More importantly, although these textbooks often have some very good group activities, their development of the material is quite conventional. This means that the standard approach is to give a mathematical definition when a new concept is introduced and then give a few examples. Then, the textbook will have a few exercises or problems that deal with this concept. The problem is that this method really does not engage the student in the development of the concept or help the student learn how to understand the concept. To understand the concept, the student must be able to develop his or her own examples related to the concept.

One problem confronting Dr. Novotny and me is how to develop course materials that do not have these drawbacks. We have begun discussions about this but have not yet reached a conclusion except that what we develop will probably not be a conventional textbook. It may end up being more like a "workbook" for the course where the students will be expected to detail much of their own work. A separate Instructor's Manual will be available which will contain hints, suggestions, and answers. Some of the material in the Instructor's Manual would be made available to students only after they have completed their own work.

It should be clear from the previous discussion that Dr. Novotny and I do not plan to produce course materials that will be in the form of a "traditional" textbook. This is consistent with the following goals that were first stated in Part 2, Goals and Objectives:

1. We will have complete instructional materials for ourselves when we teach the course. These materials will be written so that they are consistent with the guidelines that will be approved by the Department of Mathematics and Statistics.

2. These materials will be available to any faculty member teaching MTH 210 at Grand Valley State University. If possible, we would like to have an "electronic" version available to all faculty members. In this way, individual faculty members could use selected portions of the materials and could even modify them to suit their own instructional methods.

6. Arrangements with Other Institutions

There are no arrangements with other institutions. I will be completing this project at home and will be consulting with Dr. Karen Novotny at Grand Valley State University.

7. Curriculum Vita

See Appendix B.