SABBATICAL LEAVE REPORT
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Descriptive Title of the Project

Mathematical Reasoning and Communication: Exploration, Proof, and Writing.

Introduction and Background Information

I completed the primary goal of my sabbatical project, which was to develop a complete set of instructional materials for MTH 210 - Communicating in Mathematics. The end result is a preliminary version of a textbook for the course whose title is *Mathematical Reasoning: Exploration, Proof, and Writing in Mathematics*. All four sections of MTH 210 in the Fall 2000 semester and all three sections of MTH 210 in the Winter 20001 semester will be using this text. I will be teaching two of these sections in the fall and one of the sections in the winter. The Table of Contents for this text is attached to this report as Appendix A.

MTH 210 is a very important course in the mathematics curriculum at Grand Valley State University. It is a required of all mathematics majors and minors. It is a prerequisite for many of the upper level mathematics courses. Its importance, however, is not only in the mathematical content in the course. It is important because it is the first course where the focus is on the formal development of mathematics. It is the course that the department uses to teach students how to be able to read and understand mathematical definitions and proofs and how to write their results according to accepted guidelines so that their work (and reasoning) can be understood by others. This course sets the expectations for the upper level mathematics courses.

MTH 210 is also very important because it is a Supplemental Writing Skills (SWS) course. The faculty of the Department of Mathematics and Statistics have decided to have this SWS course in the mathematics major at the lower level rather than the upper level to emphasize the importance of writing in mathematics. If students are taught good (mathematical) writing habits early, we can expect better writing from students in our upper level courses. We can also reinforce the writing skills learned in MTH 210. The goal is that when students complete the mathematics major, they will have the ability to effectively write mathematical exposition. Consequently, writing guidelines are a very important part of the course.

A course such as MTH 210 is becoming a standard part the mathematics major at most colleges and universities. This course is often referred to as a "transition course" to the upper division courses in the major. The transition is from the problem solving orientation of calculus courses to the more abstract and theoretical upper level courses. This type of course is need today because the principal focus for most calculus courses is developing students' understanding of concepts and improving their problem solving skills. Consequently, most students complete their calculus courses without even seeing a formal proof or having constructed a proof of their own. However, the emphasis in many upper level mathematics courses is on the study of formal, abstract mathematics, and the expectation is that students will be able to construct proofs and write coherent, understandable mathematical proofs.

Important Features of the Textbook I Have Written
In my Sabbatical Proposal, I described problems with most textbooks that we have used for MTH 210. Following is a description of some of the important features of the text I have written and how these features address the problems of other textbooks we have used for MTH 210.

1. Emphasis on Writing in Mathematics
The course we teach is an SWS course. Most textbooks do not really address the issue of writing mathematical exposition, and so they do not address one of the primary objectives of the course. Guidelines for writing mathematical proofs are incorporated into my text. These guidelines are introduced when needed and begin in Section 2 of the text. An appendix of the text contains the complete set of guidelines for writing mathematical proofs. (These guidelines are attached to this report as Appendix D) Another problem is that most textbooks do not follow the usual conventions for writing mathematical exposition. I have attempted to write every proof in the text according to these guidelines.

2. Instruction in the Process of Constructing Proofs
One of the primary objectives of MTH 210 is to develop student's ability to construct mathematical proofs and then to write the proof in a coherent manner that conveys an understanding of the proof to the reader. These are two distinct skills, and unfortunately, most textbooks do not address the actual process of constructing and writing a proof. I have attempted to address both in the textbook I have written for MTH 210.

Instruction on how to write proofs begins in Section 2 of the text I have written and is developed further in Sections 7 through 11. Students are taught to organize their thought processes when attempting a proof with a so-called "Know-Show-Table." Students use this table to work backward from what it is they are trying to prove while at the same time working forward from the assumptions of the problem.

There are textbooks that do discuss how to construct a proof but for the most part, they do not provide any guidelines for writing the proof. The primary example of this type of text is *How to Read and Do Proofs* by Daniel Solow.

3. Mathematical Content for MTH 210
Those textbooks that deal with issues related to constructing proofs usually do not cover material that the department would like to have included in this course. During the Spring of 1998, the mathematics faculty who teach MTH 210 and upper division courses for which MTH 210 is a prerequisite agreed on the following content for MTH 210: Logic; Quantifiers with an emphasis on negations; Elementary Set Theory; Functions, including injections, surjections, and inverse functions; Elementary Number Theory including Congruence Arithmetic. This list summarizes the content of the text I have written for MTH 210.

4. Emphasis on Active Learning
The methods used to teach mathematics courses have been changing. A major emphasis now is to have the students explore mathematical concepts rather than having the text or the instructor simply tell them about these concepts. That is, the students should be actively involved in the learning process. This often involves collaborative learning where students work in groups to

brainstorm, make conjectures, test each others' ideas, reach consensus, and hopefully, to develop sound mathematical arguments to support their work. Many textbooks we have used for MTH 210 are quite traditional and simply present the material and then have the students attempt to work exercises or write proofs.

To foster active learning, I have incorporated in-class activities into each section of the text. There are some textbooks that do this. However, I have also included so-called Preview Activities for each section. A complete list of the Preview Activities is attached to this report as Appendix B.

These Preview Activities will be completed by the students prior to the classroom discussion of the section. Each student can do them individually, but it will be strongly recommended that students work in teams of two, three, or four students to complete these preview activities.

Some preview activities will review prior mathematical work that is necessary for the new section. This prior work may contain material from previous mathematical courses or it may contain material covered earlier in this text. Other preview activities will introduce new concepts and definitions that will be used when that section is discussed in class.

The purpose of the preview activities is to prepare the students for the classroom discussion of the section. The idea is that by completing these preview activities, the students will be better prepared to participate in the classroom discussion. It must be emphasized to the students that it is permissible to make mistakes on the preview activities. In fact, the place to make mistakes (and then correct them) is on the preview activities.

Before the classroom discussion on a section, at least one of the preview activities for that section will be collected and graded. Grading will not be based on whether or not everything is correct, but rather on whether or not a serious and substantial effort was made to complete the preview activities.

In addition to the Preview Activities, each section of the text contains two or three activities related to the material contained in that section. These activities can be used for in-class group work or can be assigned as homework. A complete list of these activities is attached to this report as Appendix C.

Other Goals of My Sabbatical Project

My sabbatical project had three goals. They were:

1. To develop a document to be approved by the Department of Mathematics and Statistics that will describe:

    - The objectives of MTH 210 - Communicating in Mathematics.
    - The writing requirements for this course.
    - How the writing requirements satisfy the SWS requirements as established by Grand Valley State University.
    - The types of proof techniques that will be studied in this course.

- The mathematical content that will be studied in this course.

This document will be for the faculty of the Department of Mathematics and Statistics who will teach this course and the upper level courses for which MTH 210 is a prerequisite.

2. To develop a second document, similar to the first, but intended for mathematics majors and minors who take MTH 210. In addition, this document will be useful to them in their upper level mathematics courses.

   In addition, this document would be made available to the Writing Center. It is hoped that this document would be useful to the tutors in the Writing Center as they assist students in MTH 210.

3. To develop a complete set of instructional materials for this course that will incorporate the expectations for the course that will be in the document described in the first goal.

As I have indicated, the primary goal was the third goal. This goal has been met with the completion of the textbook I have written for MTH 210 (*Mathematical Reasoning: Exploration, Proof, and Writing in Mathematics*).

A preliminary version of a document described in the first goal has been completed but has not yet been brought to the department for approval. I anticipate that the department will discuss this document during the next academic year. Once such a document is approved, it should be quite easy to complete the document describe in the second goal.

Appendix A - Table of Contents
Mathematical Reasoning
Exploration, Proof, and Writing in Mathematics