SABBATICAL LEAVE REPORT
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This is a report of my activities during my sabbatical leave for the winter semester of 1993. I received no financial renumeration during my sabbatical leave other than my salary from Grand Valley State University.

The focus of my sabbatical leave was the history and historical development of the branch of mathematics known as Galois theory. Galois theory is named in honor of the French mathematician Evariste Galois (1811-1832). He is recognized by many as one of the greatest creative mathematicians of all time. Galois' work arose out of the problem of finding solutions of polynomial equations but its importance goes far beyond the study of polynomial equations. His work laid the foundation for the development of two of the most important algebraic structures in mathematics, groups and fields.

My primary goals for this project were to further my understanding of Galois theory and to enhance enhance my appreciation for the actual development, over a long period of time, of a mathematical theory. To accomplish these goals, I developed a timetable for the winter semester of 1993 to research the historical development of methods to solve polynomial equations, the origins of Galois theory as it was developed in the early nineteenth century, and the development of Galois theory from its concrete origins to a more general and abstract theory developed in the first half of the twentieth century.

My timetable for this project was divided into seven phases, and although I did not follow the timetable exactly as designed, I did accomplish most of what I set out to do in this timetable. (A copy of this timetable is appended to the end of this report.) During the winter semester, I made several trips (usually once per week) to Western Michigan University to do library research. I also made occasional trips to the mathematics libraries at the University of Michigan and Michigan State University. As expected, during this library research, I discovered various sources for researching the development of Galois theory that I had not anticipated while I was writing my sabbatical proposal ([G1], [L2], [M1], [R2], [T1], [W1]). This is one of the reasons why my actual research did not exactly follow the timetable that I had developed for the proposal.

Another reason was the unexpected difficulty I had in studying some of the classical works of famous mathematicians of the past. I knew that working with old mathematical works would be difficult because of differing terminology, notation, and writing style. However, I did not

anticipate the difficulty I would have in understanding the mathematical writings of some of the mathematicians of the late eighteenth and early nineteenth centuries (Lagrange, Abel, and Galois). Consequently, the main part of the timetable that was not completed was Phase 3 (the study of some of the original works of some of the important predecessors of Galois). Fortunately, this did not cause a great deal of difficulty as there were several sources describing the original works of these mathematicians ([E1], [R2], [T1], [W1). In addition, during Phase 2, I had a great deal of difficulty understanding the description of the works of Abel, Gauss, and Lagrange in the primary source for that phase, "A Development of Galois Theory from Lagrange to Artin" [K1]. In hopes that it would make this work understandable for me, I decided to skip to Phase 4 and study Galois' original works as developed in the book *Galois Theory* by Edwards [E1]. This helped immensely and I was able to go back and complete Phase 2 after working through Edwards book and the book by Tignol [T1]. After this, I was able to complete Phase 5 and Phase 6 of my timetable.

Phase 7 (the development of seminars and articles based on this project) is still in progress. I am still hoping to develop a series of seminars for students and faculty describing the development of the solutions of polynomial equations, Lagrange's contributions to group theory, and the work of Galois.

Bibliography

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APPENDIX 1

The proposed timetable from my sabbatical proposal

Phase 1 (to be completed by January 1, 1993) - A general study of the history of algebra with particular attention to the development of the solutions of cubic and quartic equations in the sixteenth century.

This phase will be done prior to my sabbatical leave. It will be done by studying the book, *A History of Algebra* , by B. L. van der Waerden [**V1**]. The other main source for this phase will be *The Great Art or the Rules of Algebra* by Girolamo Cardano [**C1**]. This book was originally published in 1545 and has been translated into English. It is considered to be a systematic presentation of the methods for solving equations that were known at that time.

Although not mathematical in nature, another part of this phase is to study French. In order to study some of the original works in the next phases, I will need a reading knowledge of French. Although I passed a French language exam for my Ph.D., I have not used French since then, and I am assuming it will take a some effort to reacquire this ability.

Phase 2 (to be completed by January 10, 1993) - An overview of the development of Galois theory from the late eighteenth century to the early twentieth century.

The primary source for this phase will be the article by B. Melvin Kiernan, "A Development of Galois Theory from Lagrange to Artin." [**K1**]. This lengthy article (over 110 pages) seems to be the most detailed and exhaustive study of the development of Galois theory. It includes historical material and mathematical material that shows how the theory was developed. This article should provide a good deal of perspective that will help with the study of some original works of the great mathematicians.

Phase 3 (to be completed by February 10, 1993) - Research into and study of the original works of some of the important predecessors of Galois.

The most important predecessors of Galois were Lagrange, Gauss, and Abel. I plan to study portions of *Reflexions sur la Resolution Algebrique des Equations* by Lagrange [**L1**], *Disquisitiones Arithmeticae* by Gauss [**G2**], and *Oeuvres Completes de Niels Henrik Abel* [**A1**]. There is one important article of Abel's translated into English in *A Source Book in Mathematics* , edited by David Eugene Smith [**S2**].

Phase 4 (to be completed by March 15, 1993) - A study of Galois' original works.

The first part of this phase will be to study the book *Galois Theory* by Harold M. Edwards [**E2**]. This book was written in the spirit of Galois' original work and includes material on the work of the predecessors of Galois. The idea of the book is to show how Galois originally developed his ideas. In the preface, the author states that he wanted to "write about the theory in a way that would not only explain it, but explain it in terms close enough to Galois' own to make his memoir accessible to the reader." [**E2**, p. vii] An appendix in this book has an English translation of Galois' fundamental work, "Memoir on the Conditions for Solvability of Equations by Radicals". After studying this, I will study all portions of Galois' original works that pertain to the development of Galois theory. For this, I will use *Oeuvres Mathematiques d'Evariste Galois* [**G1**] and *Ecrits et memoires mathematiques d'Evariste Galois* [**B3**] .

Phase 5 (to be completed by April 15, 1991) - The status of Galois theory at the end of the nineteenth century.

Much of the work in Galois theory in the late nineteenth century was done in Germany rather than France, but the sources that I plan to use are articles by Bolza [**B2**] and Pierpont [**P1**], [**P2**], [**P3**], and [**P4**]. These articles were written in the tradition of the German development of Galois theory and appear to be the first articles on the theory written in English.

Phase 6 (to be completed by May 1, 1991) - The development of the modern abstract formulation of Galois theory.

The main source for this phase will be Emil Artin's book [**A2**]. Although there has been much work in Galois theory since Artin, he was the first to be credited with the developing modern, abstract formulation of Galois theory that is concerned with the relation between field extensions and groups of automorphisms. With this more abstract formulation of Galois theory, the original problem of finding algebraic solutions to polynomial equations becomes an application of the theory rather than the problem that motivated the development of the theory. By completing this phase, I will have completed my sabbatical leave and my journey through Galois theory from its inception to its modern abstract formulation.

Phase 7 (to be completed by January 1, 1994) - The development of the seminars and articles based on this project.

I am sure that some of this work will begin during the previous six phases, but most of the work will be done after the completion of the sabbatical. I plan to begin extensive work on this phase during the summer of 1993.