International Journal for Philosophy of Religion **48:** 131–138, 2000.

Chaos and providence

JASON COLWELL
California Institute of Technology, Pasadena, California, USA

Introduction

The purpose of this article is to examine a possible connection between chaos as it is conceived in contemporary mathematics and the providence of God. I propose that randomness may well be a means whereby God uses chaos to act providentially in His Creation. My intention here is primarily explanatory and not apologetical. I am not claiming that chaos theory provides evidence for the existence of God. Rather, I am presupposing that God exists and that He is omniscient, omnipotent, omnipresent and omnibenevolent. Given that God exists with the attributes classically said to belong to Him, I want to offer an explanation as to how God can use chance and chaos to act in the universe in accordance with the laws of physics. Of course my explanation can serve a negative apologetical purpose. It addresses the skeptical claim that fundamental randomness in nature is inconsistent with the existence of God; for my position is that through chaos, random events may be effective instruments of God's providence.

One example of God's possibly using chaos to bring about His will without countermanding natural law is found in the book of James. There the author records the following instance of answered prayer: "Elijah was a man just like us. He prayed earnestly that it would not rain, and it did not rain on the land for three and a half years" (5:17, NIV). Before we can see how God might have used chaos to answer Elijah's prayer, we need to lay a conceptual foundation. I shall therefore begin by explaining what I mean by the three terms "chance," "chaos," and "providence."

Chance

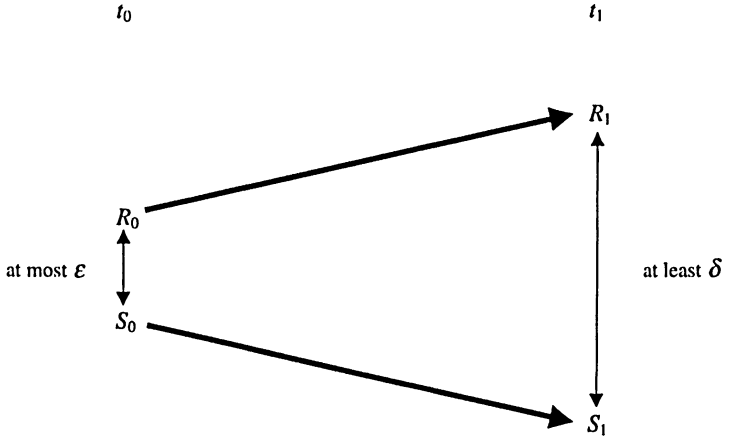
I use the term *chance* to refer to the state of nature which makes an event unpredictable. There are two types of chance to consider. The first is a state which, although it makes an event practically unpredictable, is itself determined by physical laws. The tossing of a coin provides us with an example.

The outcome of the toss is determined by the forces exerted on the coin during the toss, by the height of the toss, the size and weight of the coin, the viscosity of the air, and even by the type of surface on which the coin lands. That is, fundamentally, the outcome is determined by physical laws. Yet under normal conditions, the precise interplay of those laws is not worked out, and would in fact be very difficult to do. Thus, for all practical purpose, the toss is unpredictable.

The second type of chance refers to the state of nature which makes an event undetermined by any physical law, that is, fundamentally unpredictable. The position of an electron within an atom, for example, is apparently undetermined by anything more fundamental to the natural world. Its position over time is constrained by a *probability distribution*, which is a description of how often the electron will be within various regions of space. But the electron's location at any given time appears to be unpredictable in principle.

Chaos

"Chaos" has several possible precise mathematical definitions. What they all have in common is the following: the system must be *sensitive to initial conditions*, a property I will discuss here. A *dynamical system* is a time-varying system whose state at a point in time gives rise to its states at subsequent times. A *chaotic* dynamical system is one where an arbitrarily small variation in its state at one point can produce a significant difference in its state after enough time has elapsed. The idea can be made precise with the aid of the following diagram.



There is a set W which consists of all the possible states of the system. In the case of global weather, which is a chaotic dynamical system, W is the set of all possible states of weather over the entire globe. The set W itself does not vary with time, although the system's states, which are elements of W, do. In mathematical language, W is a metric space. (This means there is a way of measuring the difference between any two members of W.) The set W includes two states R_0 and S_0, differing by at most a small amount ε. These states, if they occurred at a time t_0, would cause the system at a later time t_1 to have, respectively, two states R_1 and S_1 (also members of the set W). The resultant states R_1 and S_1 differ from each other by a significant amount δ. To illustrate with the example of weather, R_0 and S_0 might differ by only a small current of air, whereas it might be that R_1 has a hurricane in the Caribbean while S_1 does not.

The nature of chaos lies in the fact that δ is an invariant of the system while ε can be as small as desired. The time interval $t_1 - t_0$ depends on ε. The smaller we wish ε to be, the larger the time interval required. In other words, the more similar we want the initial states, the longer it will take for those states to differ significantly. (Note that in addition to the time t_1 depending on ε, the states R_1 and S_1 (say, the location of the hurricane in R_1) which would result, depend on how small we want the air current to be. But no matter how similar we wish R_0 and S_0 to be, there are always two such states at t_0 which can give rise to significantly different ones (that is, differing by at least δ) at a later time t_1.)

A consequence of the property of chaos is the impossibility of predicting the development of such a system into the indefinite future by means of instruments of finite precision. For suppose that at t_0 we measure states of a chaotic system using instruments capable of detecting a variation only of size at least ε. Then our instruments might fail to distinguish between R_0 and S_0, differing by less than ε, which would give rise, respectively, to the significantly different R_1 and S_1 at t_1. In other words, observable occurrences in the future can result from present occurrences too minute to observe in the present.

By contrast, most mechanical systems with which we deal from day to day are predictable and hence are non-chaotic. An example of a non-chaotic system is the simple pendulum, whether an ideal, frictionless one, or a weight on a string. If two identical pendula at rest are held at different positions and released simultaneously, their subsequent positions will never differ by more that the initial ones did.

But many natural systems are unpredictable and chaotic. Examples of chaotic systems in nature include the rhythms of the human heart, eddies in a stream, and the earth's weather. More specifically, the tiny current of air

caused by the wings of a butterfly could cause a hurricane a few weeks later in another location. In fact, even a smaller initial event could suffice. The motion of a single molecule could accomplish the same drastic effect, though a longer time interval between this event and the hurricane would be required.

There are two possible misconceptions about chaos which warrant mention before leaving this section. The first is that *any* two arbitrarily similar states R_0 and S_0 could eventually produce, respectively, two states R_1 and S_1 differing by at least δ. But the difference between R_0 and S_0 may not increase over time. In other words, not every small difference in the system at one time would cause a significant difference later on. For instance, not every butterfly causes a hurricane. The second misconception is that any two states R_1 and S_1 differing by at least δ would be caused, respectively, by two very similar states differing by at most ε. This is equally false; not every hurricane could have been caused by a butterfly. So the two misconceptions are: every tiny difference would cause a big one, and every big difference could result from a tiny one.

Providence

By *providence* I mean divine action in the universe without the suspension of natural law. In a universe where physical law was entirely deterministic, providence would be impossible. In such a universe miracles could still occur, by God's suspension of the laws of physics. I am not by any means attempting to restrict God's interaction with his Creation to providence. But in a deterministic universe, every divine act would contradict the natural course of events, and would thus be a miracle, even if unnoticed by humans.

Randomness and divine choice

Mathematically we can express the idea of a random event as follows. Suppose X is a random variable with probability density function $P(X)$. Let $\{x_1, x_2, \ldots\}$ be the sequence of values of X. Then as time goes on, the set $\{x_1, x_2, \ldots, x_n\}$ of values of X which have occurred up to that time conforms to $P(X)$. Precisely,

$$\lim_{n\to\infty} \frac{|\{x_i | 1 \leqslant i \leqslant n, x_i \in R\}|}{n} = \int_R P(X)dX$$

for any subset R of the possible values of X.[1] What this means is that in the long run, the distribution of outcomes of the random process is fixed.

However, this property leaves any single outcome undetermined. A coin toss provides a good illustration here. $P(X)$ is very simple in this case.[2] It is:

$$\begin{cases} P(\text{heads}) = \frac{1}{2} \\ P(\text{tails}) = \frac{1}{2} \end{cases}$$

In other words, the probability of heads is $\frac{1}{2}$, or 50%, and the probability of tails is the same. If a coin is tossed repeatedly, the fraction of tosses which are heads will approach $\frac{1}{2}$. But this does not determine the outcome of any one toss.

I propose divine choice to be the determinant of fundamentally random events. If this is so, then using the foregoing notation, God upholds natural law by choosing values of X in accordance with the distribution $P(X)$, while at the same time having tremendous latitude in choosing any one value of X.

This proposal sees God's hand continuously holding the physical laws of the universe in place, they being essentially patterns of divine choice. It is also a way for God to work *through*, and not against, these physical laws. Natural law, so called, is at every moment the direct choice of God. It is the consistent patterns of these choices which constitute the regularity of our world.

Chaos, randomness, and providence

Chaos enables minute features of a system's present state to have significant effect upon its future state. If random events, determined by divine choice, occur within a chaotic system, God can make significant decisions about the system's future states *without countermanding natural law*. For example, He could, at the microphysical level, choose the position of an electron at one time while preserving its probability density function through His pattern of choices over all time. The electron's position at that moment could influence the motion of one, then several air molecules. This would soon affect the flow of a tiny region of air. Amplified through chaos, this could cause a significant meteorological event after more time had elapsed.

God, being omniscient, sees all the intricate workings of chaotic systems. He knows where tiny changes would have huge effects later on. This enables Him to act providentially in many situations to produce a desired result.

There are two ways the tiny changes could be produced. On the one hand, God could have created a mechanism which generates randomness. On the other hand, it might be that he chooses the outcome of each random event. I shall denote the model presented in this article by M_1 or M_2, depending on which of the two respective ways (or possibilities) is taken to be God's mode of action upon His creation.

In M_1, God has created a mechanism which generates randomness in accordance with certain probability distributions upon which He has decided. Most of the time He lets it operate without divine intervention. The mechanism is responsible for the fundamental randomness in nature: for example, the position of subatomic particles. Occasionally, God intervenes and selects a particular outcome, or small set of outcomes, of the randomness generator. This allows him to act providentially in nature. As these interventions are relatively few and scattered, the probability distributions of these random events, which are the basis of regularity in nature, are not distorted. While this model, M_1, is different from that of determinism, they share something in common. For in both, God has created a "machine" that runs on its own, which God allows it to do most of the time. In the deterministic model, the "machine" is natural law itself. When God intervenes, a miracle results. In M_1, the "machine" is the randomness generator, and natural law consists of the patterns of regularity which result from the workings of the machine. God can intervene in the workings of natural law, that is, in the causal relationship between the randomness generator and nature – this is a miracle – or, He can intervene in the operation of the machine and select certain outcomes – this is providence. M_1 gives God somewhat greater latitude in the way He can work in nature than does determinism. But there remains the similarity between the two. Though the machine in M_1 is not natural law itself, it is still a created mechanism which operates independently of God.

I favour M_2, the second way of accounting for the tiny changes which have huge effects at a later time. I am proposing divine choice to be the determinant of *each* fundamentally random event. Thus, as stated earlier, God's hand continuously upholds physical laws by making the fundamentally random events conform to the distributions upon which He has decided. This version of my model allows God to act in His creation without having to disrupt any created mechanism. God can act while leaving natural law (observable patterns in nature) undisturbed, and without interfering, as in M_1, with a randomness generator. According to M_2, the tiny changes are simply (unobservable) divine choices which have observable effects later. This does not preclude the possibility of miracles, which is discussed in the following section.

The dominant and unifying feature of my model is God's constant volitional involvement with His creation. M_2 exemplifies this feature more richly than does either determinism or M_1. In the type of determinism we have been considering, God acts upon His creation by interference with an independently operating universe. In M_1 God moves somewhat closer to His creation by acting upon the universe through small adjustments He makes to a randomness generator. But the greatest intimacy with his creation is displayed

in M_2 as God continuously makes active decisions about each fundamentally random event in the universe. Not only does M_2 present the clearest picture of God's intimate involvement with his creation but it also exemplifies sharply the divine attribute of omnipresence. Moreover, M_2 comports well with biblical theology. We are told in Matthew 10:29–30 that all the hairs of our head are numbered and that no sparrow falls to the ground apart from God's will. Furthermore, in Acts 17:28 the apostle Paul essentially says to the Athenian philosophers that God's presence suffuses His creation, "[f]or in him we live and move and have our being."

A contrast with miracle

As stated earlier, though I am proposing that God can act in the universe without disrupting the regularities of nature, I am not discounting the occurrence of miracles. I wish to illustrate the difference in M_2 between His acting providentially and acting miraculously. The book of Exodus, chapter 14, records God's parting of the Red Sea to enable the Israelites to escape the clutches of the pursuing Egyptian army. The "walls" of water on both sides of the dry seabed could not have been kept in place be the "strong east wind" acting unaided, or by anything consistent with normal weather and normal gravitational and magnetic forces acting on water molecules. He did not simply act by making particular choices of the outcomes of fundamentally random events while preserving the normal distribution of outcomes of those events. God might have held the electrons of the water molecules in such close proximity to neighbouring water molecules that great attractive forces within the water were created, holding the water back. But this would hardly be consistent with the usual probability density functions for the positions of the electrons. Or He might have introduced some force entirely foreign to nature to restrain the water. In either case, God would have countermanded natural law by making a choice or choices inconsistent with His usual patterns of choice for nature.

Does the claim that natural laws are merely patterns of divine choice undermine the notion of miracle? For a miracle is normally considered to be a suspension of the laws of nature, and there must be something to suspend. Do we not want to consider God to be acting *against* something, instead of just acting differently? This objection can be answered, I think, if we note that so-called natural laws are only formulations of observed patterns in nature. Miracles are violations of natural law. Then what are miracles but disruptions of the patterns we observe in nature? When God makes a choice which deviates from His usual pattern of choices, He suspends that pattern which we call natural law.

God, chaos, and Elijah's prayer

The following is a description of the way the mechanisms discussed in this article could have been involved in the biblical account with which our discussion began. This speculative example illustrates how the process of providence might have been achieved by the determination of chance events whose effect is amplified by chaos.

Elijah prayed that it would not rain. God heard his prayer, wished to grant his request, and accordingly chose the position of some subatomic particles somewhere in the Earth's atmosphere. (This was entirely within the bounds of physical law.) The movement of some air molecules was thereby determined. Because of the chaotic nature of the earth's weather, this in time had a large-scale effect, causing a drought in Israel. Through His constant choosing of the positions of subatomic particles, God prolonged the drought, and it did not rain on the land for three and a half years.

Acknowledgments

I am grateful to an anonymous referee and to Gary Colwell for their helpful comments.

Notes

1. $|\{x_i | 1 \leqslant i \leqslant n, x_i \in R\}|$ is the number of values of X, up to the nth occurrence of the random event, which lie in the set R. Then $\frac{|\{x_i | 1 \leqslant i \leqslant n, x_i \in R\}|}{n}$ is the proportion of these values of X which lie in the set R. As the number n of occurrences increases, this proportion approaches (the meaning of $\lim_{n \to \infty}$) the probability of any particular occurrence of the event being in R. The probability is, by definition of $P(X)$, the integral $\int_R P(X)dX$.
2. This example is of a discrete, instead of continuous, random variable. If we wished to apply the above formula we could do so by replacing the integration with a summation.

References

Devaney, Robert R. (1986), *An Introduction to Chaotic Dynamical Systems*. Menlo Park, CA: Benjamin-Cummings.

Gleick, James (1987), *Chaos: Making a New Science*. New York: Viking.

Address for correspondence: Jason Colwell, California Institute of Technology, Department of Mathematics, Caltech 253-37, 1200 E. California Boulevard, Pasadena, CA 91125, USA
Phone: (626) 395-4335; E-mail: colwell@its.caltech.edu