DEVELOPING MATHEMATICAL PRACTICES FOR AP CALCULUS

GVSU MATH IN ACTION

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AMANDA (MANDY) FORSLUND

FORSLUNA@GVSU.EDU

OUR GOALS TODAY

- EXPERIENCE TEACHING STRATEGIES TO DEVELOP THE MPACS IN STUDENTS
- USE TWO FREE RESPONSE QUESTIONS FROM THE 2016 AP TEST AND
- DISCUSS HOW COLLEGE BOARD’S RUBRICS PROMOTE THE MPACS

MATHEMATICAL PRACTICES FOR AP CALCULUS

- MPAC 1: REASONING WITH DEFINITIONS AND THEOREMS
- MPAC 2: CONNECTING CONCEPTS
- MPAC 3: IMPLEMENTING ALGEBRAIC/COMPUTATIONAL PROCESSES
- MPAC 4: CONNECTING MULTIPLE REPRESENTATIONS
- MPAC 5: BUILDING NOTATIONAL FLUENCY
- MPAC 6: COMMUNICATING
  - MOST HAVE A TECHNOLOGY COMPONENT
  - EMPHASIS ON THE PRACTICE OF “DOING” MATHEMATICS
  - INTENDED TO HELP GUIDE TEACHERS TO WHAT PRACTICES ARE ESSENTIAL FOR DEVELOPING CONCEPTUAL UNDERSTANDING IN CALCULUS

(STUDENT ENGAGEMENT STRATEGY #1)

VISIBLE THINKING

WHAT DO YOU OBSERVE?

WHAT DO YOU THINK?

WHAT DO YOU WONDER?

SCORING EXPOSURE (STUDENT ENGAGEMENT STRATEGY #2) AP CALCULUS AB/BC 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

SPEED DATING (STUDENT ENGAGEMENT STRATEGY #3) AP CALCULUS AB/BC 3

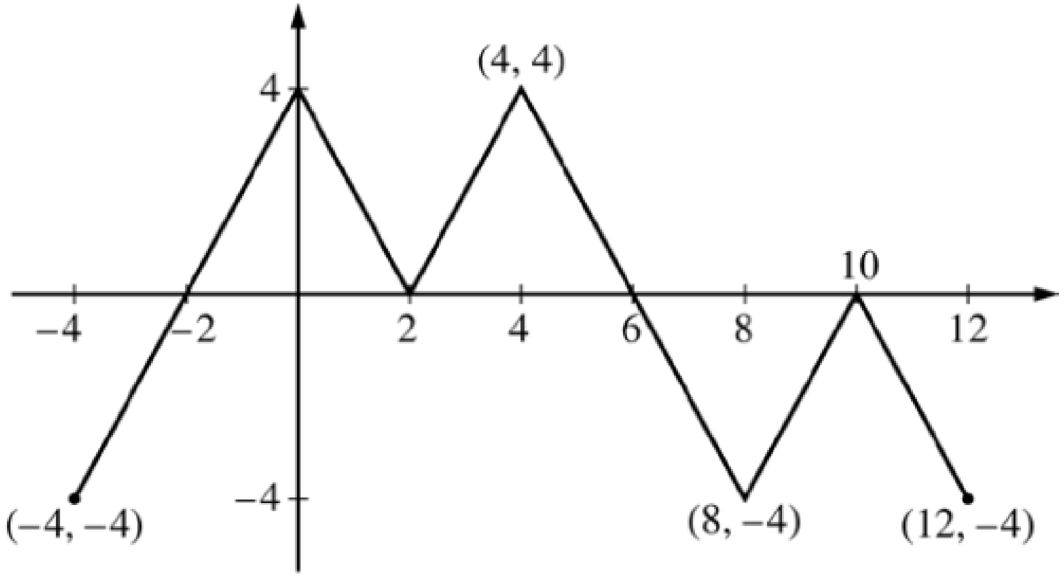
The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.



Graph of f

NEXT STEPS…

- MORE IDEAS TO ENGAGE STUDENTS IN AP QUESTIONS?
- OTHER STRATEGIES TO PROMOTE MATH PRACTICES?

Mathematical Practices for AP Calculus (MPACs)

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. They are drawn from the rich work in the National Council of Teachers of Mathematics (NCTM) Process Standards and the Association of American Colleges and Universities (AAC&U) Quantitative Literacy VALUE Rubric. Embedding these practices in the study of calculus enables students to establish mathematical lines of reasoning and use them to apply mathematical concepts and tools to solve problems. The Mathematical Practices for AP Calculus are not intended to be viewed as discrete items that can be checked off a list; rather, they are highly interrelated tools that should be utilized frequently and in diverse contexts.

The sample items included with this curriculum framework demonstrate various ways in which the learning objectives can be linked with the Mathematical Practices for AP Calculus.

The Mathematical Practices for AP Calculus are given below.

MPAC 1: Reasoning with definitions and theorems

Students can:

a. use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;

b. confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;

c. apply definitions and theorems in the process of solving a problem;

d. interpret quantifiers in definitions and theorems (e.g., “for all,” “there exists”);

e. develop conjectures based on exploration with technology; and

f. produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

MPAC 2: Connecting concepts

Students can:

a. relate the concept of a limit to all aspects of calculus;

b. use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

c. connect concepts to their visual representations with and without technology; and

d. identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

a. select appropriate mathematical strategies;

b. sequence algebraic/computational procedures logically;

c. complete algebraic/computational processes correctly;

d. apply technology strategically to solve problems;

e. attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

f. connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

a. associate tables, graphs, and symbolic representations of functions;

b. develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;

c. identify how mathematical characteristics of functions are related in different representations;

d. extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

e. construct one representational form from another (e.g., a table from a graph or a graph from given information); and

f. consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency

Students can:

a. know and use a variety of notations (e.g., $f'(x), y', \frac{dy}{dx}$);

b. connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);

c. connect notation to different representations (graphical, numerical, analytical, and verbal); and

d. assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

a. clearly present methods, reasoning, justifications, and conclusions;

b. use accurate and precise language and notation;

c. explain the meaning of expressions, notation, and results in terms of a context (including units);

d. explain the connections among concepts;

e. critically interpret and accurately report information provided by technology; and

f. analyze, evaluate, and compare the reasoning of others.

Question 1

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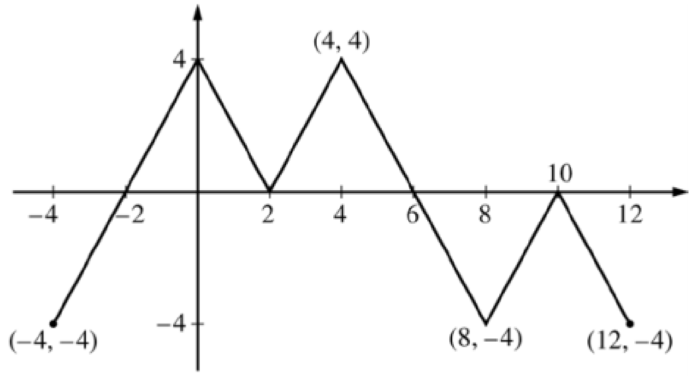
$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

Question 3



Graph of f

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(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

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(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

Question 3

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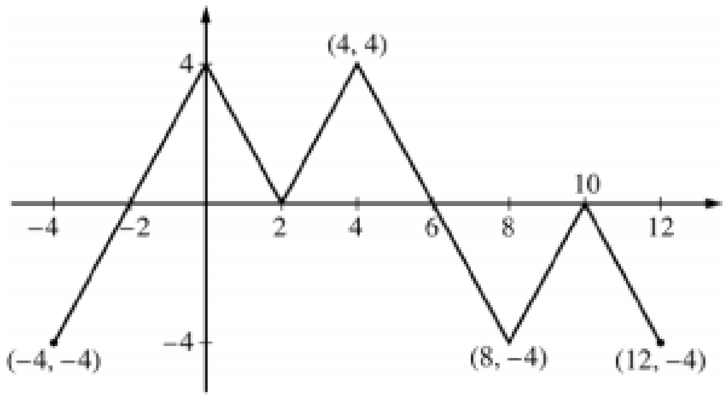

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1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

(a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \le 0$ for $8 \le x \le 12$.

1 : answer with justification

(b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

1 : answer with justification

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
−4	−4
−2	−8
6	8
12	−4

On the interval $-4 \le x \le 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

$$4 : \begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad\ \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{cases}$$

(d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$.

2 : intervals