Conway's Rational Tangle Dance Activity

(1) ***Initial Position***

Arrange four people in positions A, B, C, and D holding two ropes (about 10 feet long) as shown below. It is important that the two ropes are parallel to each other and parallel to the front of the classroom.

(2) ***TWIST, ROTATE, DISPLAY***

Only the following two moves are allowed for the dance (each can be done multiple times, in any order), either "twist 'em up" or "turn 'em around":

- TWIST: applies only to dancers in positions C and D; C and D swap places with C holding the rope up high and then D steps under it.
- ROTATE: each dancer moves one position clockwise.

The ropes are now tangled by this dance.
Show the audience what the ropes look like:

- DISPLAY: the dancers raise their ropes (back higher than front) to show the audience the tangle (*audience claps in awe*)

(3) ***TWIST***

Let's investigate what TWIST does mathematically so we can assign a number to each tangle. As we TWIST, the ropes get more and more tangled. Note: There is no untwist command.

Let T = TWIST

- Initial start position is the two parallel ropes = 0
- TWIST: +1 (increment by one)

(4) ***Group Work: Getting Back to 0***

- Form groups of ***three.***
- Use one pair of smaller ropes for two people in the group, the third person will be the recorder to take notes on your findings.
- Hold the smaller ropes parallel to the front of the classroom (initial position 0).
- Tasks:
  - (a) Determine a sequence involving T and R to undo <u>one</u> TWIST to get back to 0.
  - (b) What would the sequence be to undo <u>two</u> TWISTS?
  - (c) How would you undo <u>three</u> TWISTS?
  - (d) Determine how to undo T^4 and then T^5.
  - (e) If ***n*** TWISTS are performed, derive a formula using T and R to get back to 0. Record your results below.

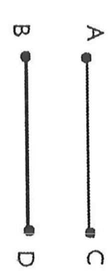
	Number of TWISTS	**Formula to get back to 0**
T	1	
$T\,T = T^2$	2	
$T\,T\,T = T^3$	3	
$T\,T\,T\,T = T^4$	4	
$T\,T\,T\,T\,T = T^5$	5	
n T's in a row = T^n	***n***	

(5) ***ROTATE***

Next, let's try to assign a mathematical operation to R = ROTATE to see what it does to the tangle.

- New dance: Start at 0
  R
  DISPLAY

We get this configuration:



(6) *More Complicated Tangles*

Question: If we perform a sequence of twists and rotates, is it always possible to get back to zero (that is, our initial position of parallel ropes)?

(a) *A Messy Tangle*:

Four new participants will create the following more involved tangle, then DISPLAY it for the entire group:

$$TTT\,RTTTT\,RTT$$

$$or$$

$$T^3RT^4RT^2$$

Task: Write the sequence of T's and R's, with associated tangle values, required to get back to the zero (0) position (initial parallel ropes). Note: Each time a T is performed, the value of the tangle goes *away* from zero. What are some other observations?

(b) *Coolest* Tangle:

- Have the same four participants create the tangle:

$$TTTT\,RTTTTT\,RTTT$$

$$or$$

$$T^4RT^5RT^3$$

- DISPLAY the tangle
- Tie a plastic bag over the entire tangle (knot)
- Work through the math to undo it
- Remove the plastic bag – *Voila*!

(7) Conway's Rational Tangles: Michigan Math Learning Standards

CCSS.Math.Content.6.EE.A.2
Write, read, and evaluate expressions in which letters stand for numbers.
CCSS.Math.Content.6.EE.A.2.a
Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 - y.

CCSS.Math.Content.7.EE.B.4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

CCSS.Math.Content.8.F.A.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

CCSS.Math.Content.HSF.BF.A.1
Write a function that describes a relationship between two quantities.
CCSS.Math.Content.HSF.BF.A.1.a
Determine an explicit expression, a recursive process, or steps for calculation from a context.
CCSS.Math.Content.HSF.BF.A.1.b
Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Additional References for Conway's Rational Tangles

John H. Conway, a contemporary mathematician (born in Liverpool, England in 1937) developed the fascinating "rational tangle dance" in 1967 to explain the ideas behind his work in knot theory. Conway's famous mathematical contributions are in the fields of number theory, group theory, game theory, knot theory, and combinatorics. He is known as a "playful genius" and is equally famous for creating countless games and puzzles related to mathematics.

The following links provide references to biographical material about John Conway and further information about rational tangles.

(a) http://www.wired.com/2015/09/life---games---playful---genius---john---conway/

(b) http://turnbull.mcs.st---and.ac.uk/~history/

(c) http://www.geometer.org/mathcircles/tangle.pdf

(d) http://www.mathteacherscircle.org/assets/session---materials/JTantonRationalTangles.pdf

(e) http://www.mathteacherscircle.org/assets/session---materials/ARodinTabletopTangles.pdf