# One Little Spark: Encouraging Creativity and Perseverance

Math in Action ~ Making Math Meaningful

Grand Valley State University
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*One little spark of inspiration is at the heart of all creation**

Invent, Design, Imagine, Discover are "sparked" by creativity and perseverance. Yet in the classroom with its reality of standards, textbooks, and testing creativity is often discouraged, viewed as beyond the reach of children, while perseverance is sidelined in favor of "curriculum coverage." Both creativity and persistence are essential to learning and critical to in the development of deep understanding of mathematical concepts.

*The moving power of mathematical invention is not reasoning but imagination.*

Augustus de Morgan (1866)

From the theme song for the *Journey into the Imagination with Figment* attraction at Epcot, Walt Disney World ® Resort.

***"We have known for some years now…that most children's mathematical journeys are in vain because they never arrive anywhere, and what is perhaps worse is that they do not even enjoy the journey."***

Whitcombe, A. (1988). Mathematics Creativity, imagination, beauty. *Mathematics in School, 17,* 13–15.

# Today's Mathematics Classroom

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**Vision:** a classroom where "students **confidently engage in complex mathematical tasks…draw on knowledge** from a wide **variety of mathematical topics**, sometimes **approaching the same problem from different mathematical perspectives or representing the mathematics in different ways** until they find methods that enable them to make progress"

(NCTM, 2000)

1 2 4 5

# Today's Mathematics Classroom

**Reality for many: time spent watching** as mathematical methods were demonstrated; where time was spent **committing to memory facts and algorithms** (Pehkonen, 1997), **conceiving mathematics as "a digestive process** rather than a creative one" (Dreyfus and Eisenberg, 1996), conveying instead **the belief** that math is divided into right and wrong and **that "the essence of math is getting the right ones**" (Ginsburg, 1996, Balka, 1974).

# Dan Meyer's: Math Class Needs A Makeover

Five symptoms that you're developing math reasoning skills wrong. Your students…

1. Lack initiative
2. Lack perseverance
3. Lack retention
4. Are adverse to word problems
5. Are eager for a formula

Two and Half Men approach to learning math:

- Teaching in small, "sitcom sized problems that wrap up is 22 minutes, 3 commercial breaks and a laugh track" resulting in **Impatient Problem Solvers**

# Essence of Mathematics

- “The essence of mathematics is not producing the correct answers, but thinking creatively” (Ginsburg, 1996).

- Accuracy is important as the students’ responses must fit the context of the problem and be mathematically correct

- BUT, “if we reject original or clever applications with small errors in application we discourage students from risk taking” (Poincaré, 1913).

# Creativity's Vital Role in Mathematics

- "Modern day commerce has no use for pupils graduating from school who have been **trained to mechanically solve problems in exactly one pre-given way, i.e. like a machine**"

  (Köhler, 1997)

- For individuals to use mathematics in ways that **go well beyond what they are taught creative mathematical thinking is key**. (Sternberg, 1996)

# Attempts to Define Mathematical Creativity

- One view "includes the ability to **see new relationships** between techniques and areas of application, and to **make associations between possibly unrelated ideas**" (Tammadge as cited in Haylock, 1987).

- Krutetskii approaches creativity from a problem-solving perspective characterizing creativity in the context of **problem-formation (problem-finding), invention, independence and originality** (Krutetskii, 1976; Haylock, 1987).

- Others have applied the **concepts of fluency, flexibility and originality** to mathematics. (Tuli, 1980; Haylock, 1997; Kim, Cho, Ahn, 2003).

# *Criteria for Measuring Creative Ability in Mathematics* *(Balka, 1974)*

The ability to

- Formulate mathematical hypotheses
- Determine patterns
- Break from established mind sets to obtain solutions in a mathematical situation
- Sense what is missing and ask questions
- Consider and evaluate unusual mathematical ideas, to think through the consequences from a mathematical situation (divergent)

# A More Realistic Problem

http://blog.mrmeyer.com/2016/3acts-nissan-girl-scout-cookies/

# Dan's Recommendation

1. Use multimedia to bring the real world into the classroom
2. **Encourage student intuition**
3. Ask the shortest question you can
4. **Let students build the problem**
5. **Be less helpful**

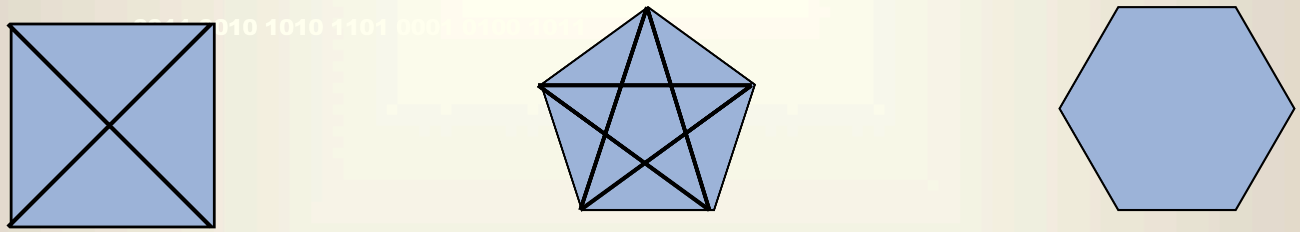
# An Answer
# (not the one and only “Right” one)

# *A Criteria for Measuring Creative Ability in Mathematics* (Balka, 1974)

The ability to:

- **Formulate mathematical hypotheses** concerning cause and effect in a mathematical situation (divergent)
- **Determine patterns** in mathematical situations (convergent)

# What is your hypotheses?



**Sample Problem**: Is there a connection between the number of sides of a polygon and the number of diagonals you can make?

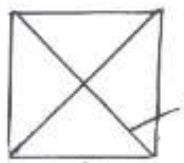
Problem is loosely structured – a research question

Cause: change in number of sides
Effect: change in number of diagonals

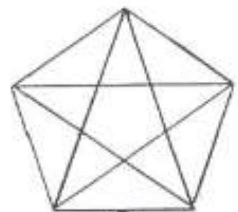
# INVESTIGATION!

EMMA

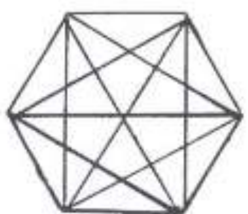
Challenge: Is there a connection between the number of sides of a polygon and the number of diagonals you can make?



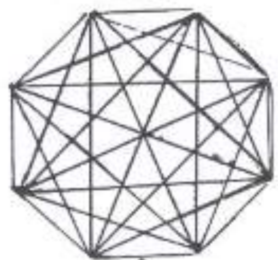

SQUARE
4 SIDES



PENTAGON
5 SIDES

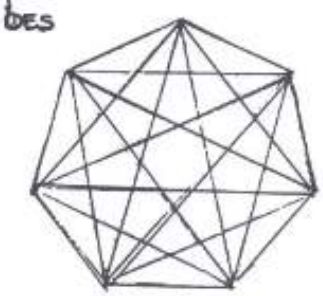


HEXAGON
6 SIDES



OCTAGON
8 SIDES

I notice that the number of sides minus 3 is number of diagonals coming of each vertex



HEPTAGON
7 sides

| NAME OF SHAPE | NUM OF SIDES | NUM OF DIAGONALS |
|---|---|---|
| QUADRILATERAL | 4 | 2 |
| PENTAGON | 5 | 5 |
| HEXAGON | 6 | 9 |
| HEPTAGON | 7 | 14 |
| OCTAGON | 8 | 20 |

I noticed that the number of diagonals of a 4, 5, 6, 7, 8 sided shape increase in a pattern (2 + 3 = 5 + 4 = 9 + 5 = 14, + 6 = 20)

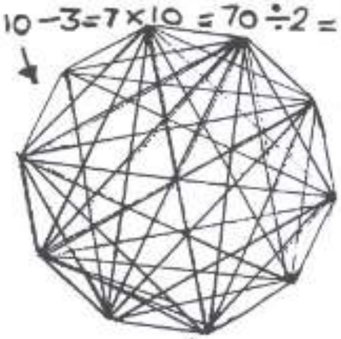
## THE RULE

I think its sides − 3 × $\frac{\text{sides}}{2}$ = diagonal

eg: Pentagon 5 − 3 = 2 × 5 = 10 ÷ 2 = amount of diagonals: 5

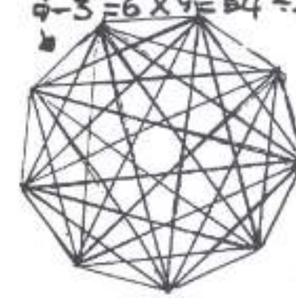
This works with any shape

10 − 3 = 7 × 10 = 70 ÷ 2 = 35



DECAGON
10 sides

9 − 3 = 6 × 9 = 54 ÷ 2 = 27



NONAGON
9 sides

I can make a formula from what I know. This formula is:

FORMULA

$\left(S - 3 \times \frac{S}{2} = d\right)$

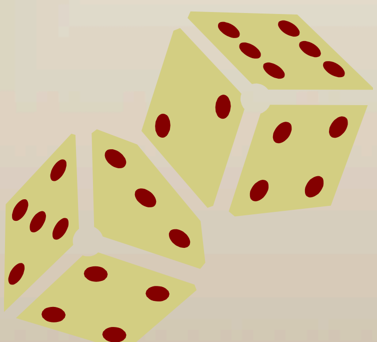
KEY
S = sides
d = diagonals
— = Halved

# What's your hypotheses?

## Roll 'em

from Awesome Math Problems for Creative Thinking – Creative Publications

- You have two number cubes, numbered 1, 2, 3, 4, 5, 6.
- Pretend you roll the cubes once and multiply the numbers on the top faces.
- What is the probability that the product would be an even multiple of 3?



# Roll 'em

- Describe the strategy you used to solve the problem.
  - What strategy(s) might your students have used?
- Would your strategy work if the die had 8 sides?
  - 10 sides? 12 sides? 20 sides? *n* sides?

# What answers you would expect from your students?

**Which answer would you accept? 15, 8, 42%, 22%, _____?**

**Is there one *"right"* answer?**

**Even multiples of 3 {6, 12, 18, 24, 30, 36}**

| | | |
|---|---|---|
| **6** | **1 x 6, 6 x 1, 2 x 3, 3 x 2** | **4 ways** |
| **12** | **2 x 6, 6 x 2, 4 x 3 3 x 4** | **4 ways** |
| **18** | **3 x 6, 6 x 3** | **2 ways** |
| **24** | **4 x 6, 6 x 4** | **2 ways** |
| **30** | **5 x 6, 6 x 5** | **2 ways** |
| **36** | **6 x 6** | **1 way** |

**There are 36 possible products, 15 of these are even multiples of three.**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | 6 |
| 2 | | | 6 | | | 12 |
| 3 | | 6 | | 12 | | 18 |
| 4 | | | 12 | | | 24 |
| 5 | | | | | | 30 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 |

# Another Challenge

- You have three jugs A, B, and C
- The problem is to find the best way of measuring out a given quantity of water, using just three jugs.
- You are told how much each jug holds.
- There are no marks on the jugs so the only way to make an accurate measurement is to fill a jug to the brim.

# Your Turn

| | Measure out | Jug A holds | Jug B holds | Jug C holds | Solution |
|---|---|---|---|---|---|
| 1 | 52 units | 10 | 64 | 1 | |
| 2 | 14 units | 100 | 124 | 5 | |
| 3 | 3 units | 10 | 17 | 2 | |
| 4 | 100 units | 21 | 127 | 3 | |
| 5 | 20 units | 23 | 49 | 3 | |
| 6 | 5 units | 50 | 65 | 5 | |

# Your Turn

| | Measure out | Jug A holds | Jug B holds | Jug C holds | Solution |
|---|---|---|---|---|---|
| 1 | 52 units | 10 | 64 | 1 | **B – A – 2C** |
| 2 | 14 units | 100 | 124 | 5 | **B – A – 2C** |
| 3 | 3 units | 10 | 17 | 2 | **B – A – 2C** |
| 4 | 100 units | 21 | 127 | 3 | **B – A – 2C** |
| 5 | 20 units | 23 | 49 | 3 | **B – A – 2C** |
| 6 | 5 units | 50 | 65 | 5 | **B – A – 2C** |

# *A Criteria for Measuring Creative Ability in Mathematics* (Balka, 1974)

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# The Einstellung Effect

- The Einstellung effect: set successful procedure applied consistently even when it is less than efficient or inappropriate
- The jug problem with 250, 11-12 year olds
  - 70% used the same procedure on all 6 problems
  - 11% used a different procedure on 1 problem
  - 11% used a different procedure on 2 problems
  - 8% couldn't do the problems

(Haylock, 1985)

# Incubation Time and Einstellung Effect

- **The cheap-necklace problem experiment (Silveira 1971)**
  "You are given four separate pieces of chain that are each three links in length. It costs 2¢ to open a link and 3¢ to close a link. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15¢."

| Control | Group 1 | Group 2 |
|---|---|---|
| • Worked for ½ hour<br>• 55% solved the problem | • Worked for ½ hour then ½ break<br>• 64% solved the problem | • Worked for ½ hour then 4 hour break<br>• 85% solved the problem |

- Students often choose particular method to solving a problem
- If not appropriate, they may be stuck with the method
- Taking a break may allow other methods a chance or students to gain a deeper understanding of the problem.

# Try This One ☺

One acre of good soil usually contains about three million worms.

How many worms can Asa and David expect to find between each 10-yard line as they dig up the perfectly manicured grass on the junior high school football field?

(Note: 1 acre = 4, 840 square yards)

(Kleiman and Washington, 1996)

# The Answer is…

- **NOT ENOUGH INFORMATION!**
- You need to know the area of the football field but the problem doesn't give you its width
- Divide area in square yards by yard per acre then find 1/10 of that, and then multiply by 3 million worms per acre
- If you knew a football field was 55 yards wide the answer is 340,909.1 worms
- Now the question is what is 0.1 worm?

# *Criteria for Measuring Creative Ability in Mathematics* *(Balka, 1974)*

The ability to

- ✓ Formulate mathematical hypotheses
- ✓ Determine patterns
- ✓ Break from established mind sets to obtain solutions in a mathematical situation
- **Sense what is missing from a given mathematical situation and to ask questions that will enable one to fill in the missing mathematical information (divergent)**

# *Criteria for Measuring Creative Ability in Mathematics* *(Balka, 1974)*

The ability to

- ✓ Formulate mathematical hypotheses
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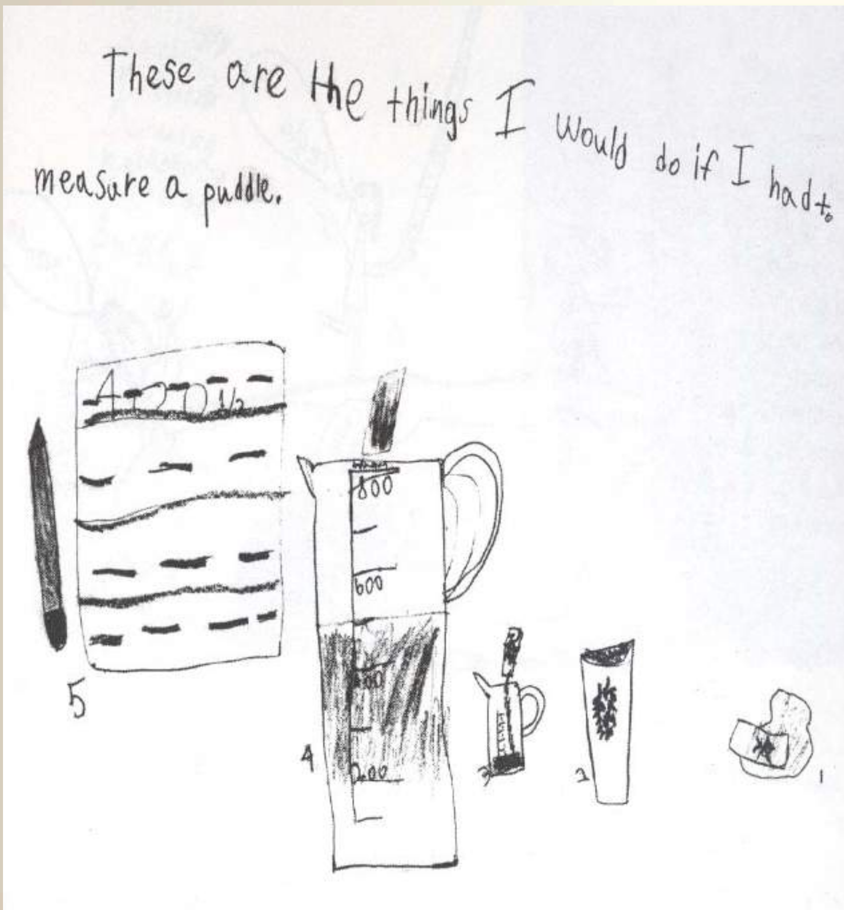
- ✓ Sense what is missing ask questions
- **Consider and evaluate** unusual mathematical ideas, to **think through the consequences** from a mathematical situation (divergent)

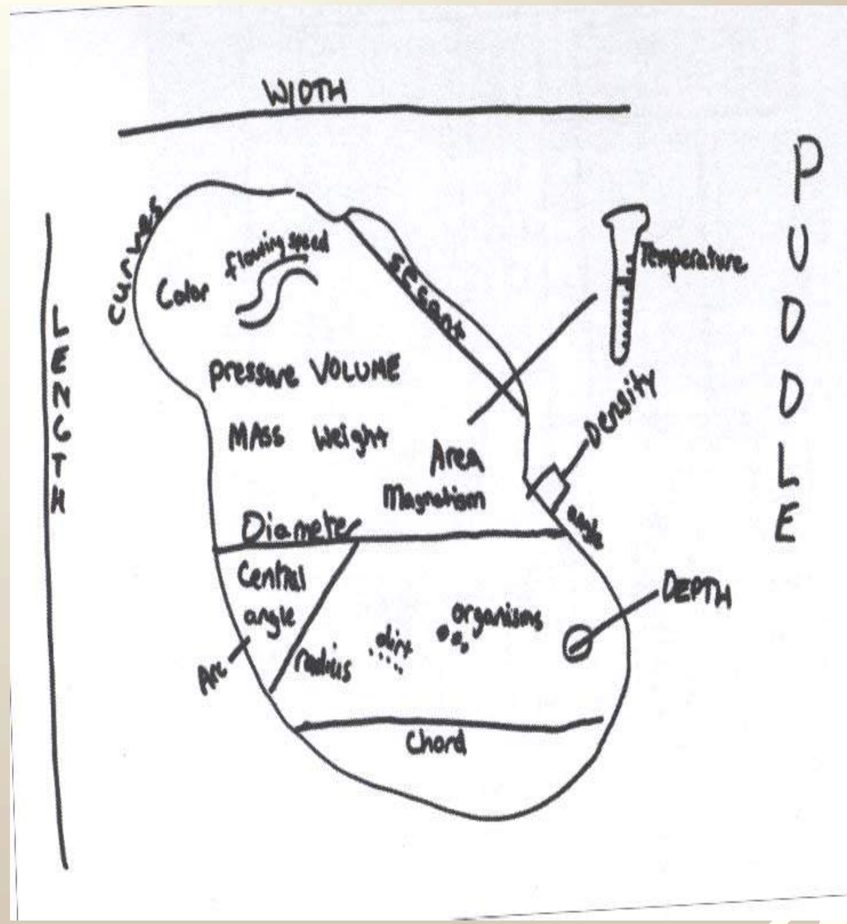
# How would you measure a puddle?

Puddle Questions – Creative Publications Grades 1-8

- Record all the different ways you can think of.
- Make sketches to show your work.

# Here are some ways by a 2nd grader…&…a 7th grader

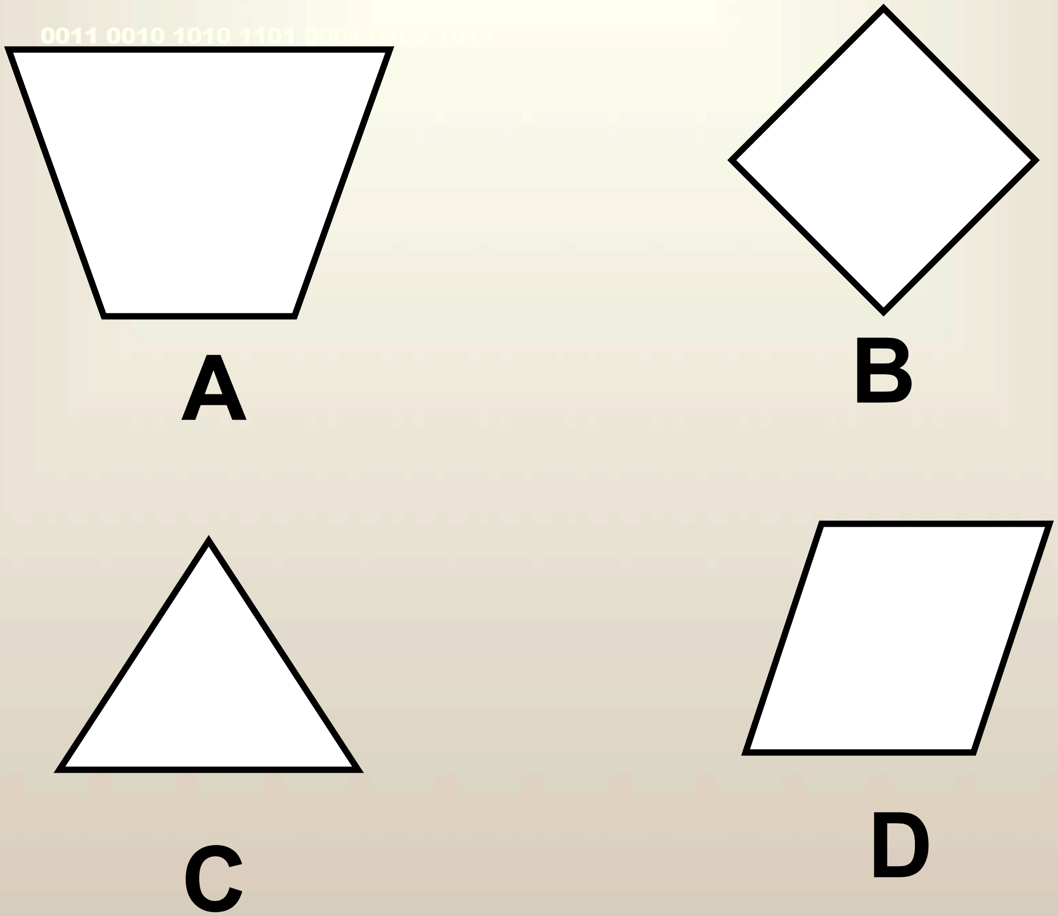


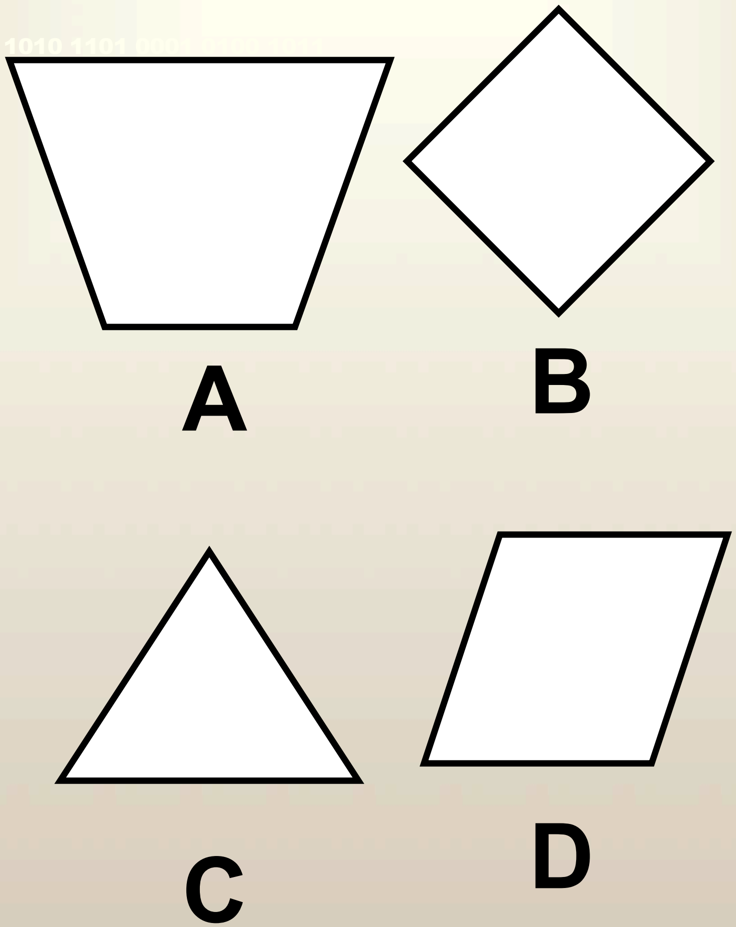

(Westly, 1994,1995)

# Which One Doesn't Belong?
# Christopher Danielson

https://www.stenhouse.com/content/which-one-doesnt-belong



# Which shape does not belong and why?




**Only one with sides of unequal length**

**Only one with right angles**

**Only one:**

- **with three sides**
- **without two sides parallel**
- **sum of interior angles = 180**

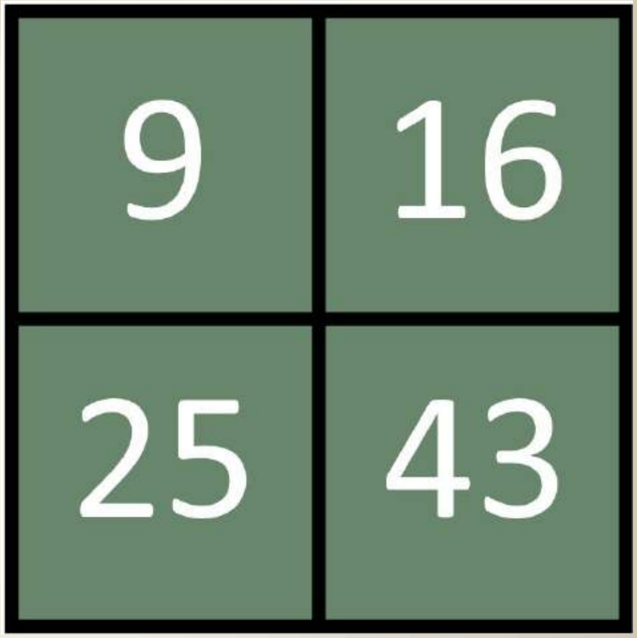
**Only one that has something in common with other shapes.**

- **equal sides D & C, B**
- **acute angles D & A, C**

Which One Doesn't Belong?

# Not just shapes

http://wodb.ca/numbers.html




# *Criteria for Measuring Creative Ability in Mathematics* *(Balka, 1974)*

The ability to

- ✓ Formulate mathematical hypotheses
- ✓ Determine patterns
- ✓ Break from established mind sets to obtain solutions in a mathematical situation
- ✓ Sense what is missing ask questions
- ✓ Consider and evaluate unusual mathematical ideas, to think through the consequences
- **Split general mathematical problems into specific sub-problems** (divergent)

# Pólya's: Let Us Teach Guessing

https://vimeo.com/48768091

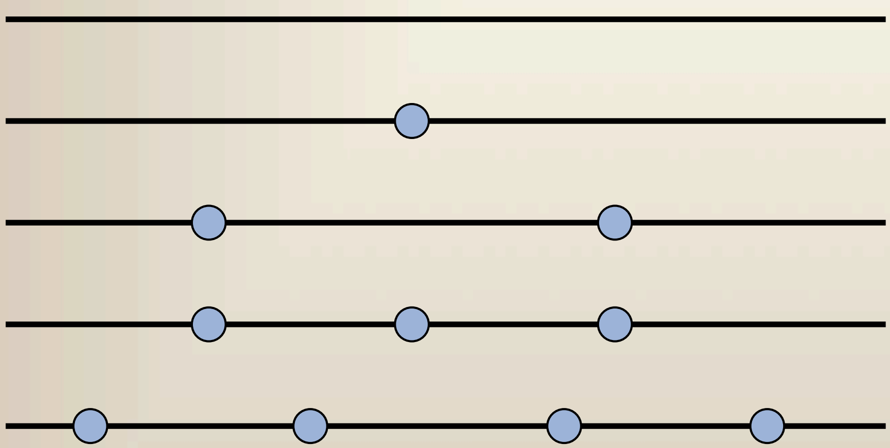
0011 0010 1010 1101 0001

- *Mathematics when it is finished, complete, all done, then it consists of proofs. But, when it is discovered, it always starts with a guess. . .*
- The problem: Into how many parts is space divide by 5 randomly intersecting planes?
- What's your guess?

1 2 4 5

# Five Planes Problem: Start with a simpler problem

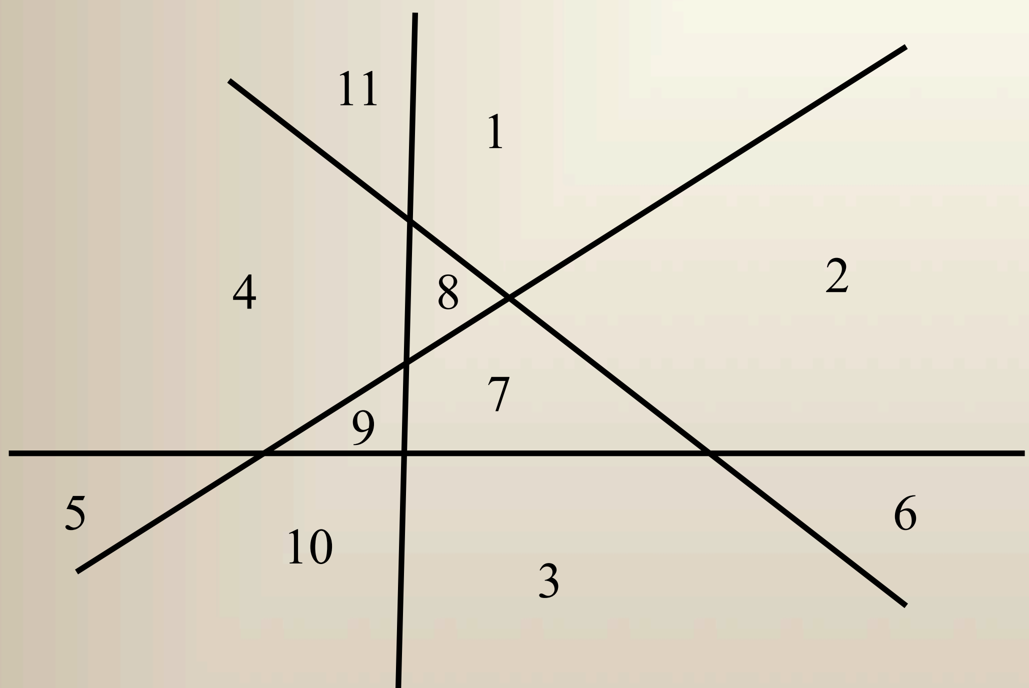
- Line/point



| *n* (point) | Space (line) |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |
| 4 | 5 |
| 5 | |

# Five Planes Problem:
# Start with a simpler problem

- Plane/line




| *n* (line) | Spaces (plane) |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 7 |
| 4 | 11 |
| 5 | |

# The Five Plane Problem

| *n* | SPACE | PLANE | LINE |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 4 | 3 |
| 3 | 8 | 7 | 4 |
| 4 | 15 | 11 | 5 |
| 5 | | | 6 |

3 + 4= 7
4 + 7 = 11

**http://www.maa.org/columns/launchings/launchings_12_07.html**

# Math in Strange Places

At the local grocery store bulletin board, Asa and David found these 8 signs. Create three problems of your own based on these signs.

***Are you Illiterate? If so, call 1-800-CANT-READ***

***Weight Watchers: $14 All You Can Eat Hot Fudge Sundae and Cheesecake Dinner***

***Time Travelers' Meeting: 7:45 p.m. last Tuesday***

***Antique Collectors Display: Pentium Computers, Then and Now. See our collection dating all the way back to 1995***

***PSYCHICS' MEETING: You Know Where, You know When***

***TIME MANAGEMENT WORKSHOP for people who find themselves just too busy. Meeting 6:30 – 9:30 weekly, attendance mandatory***

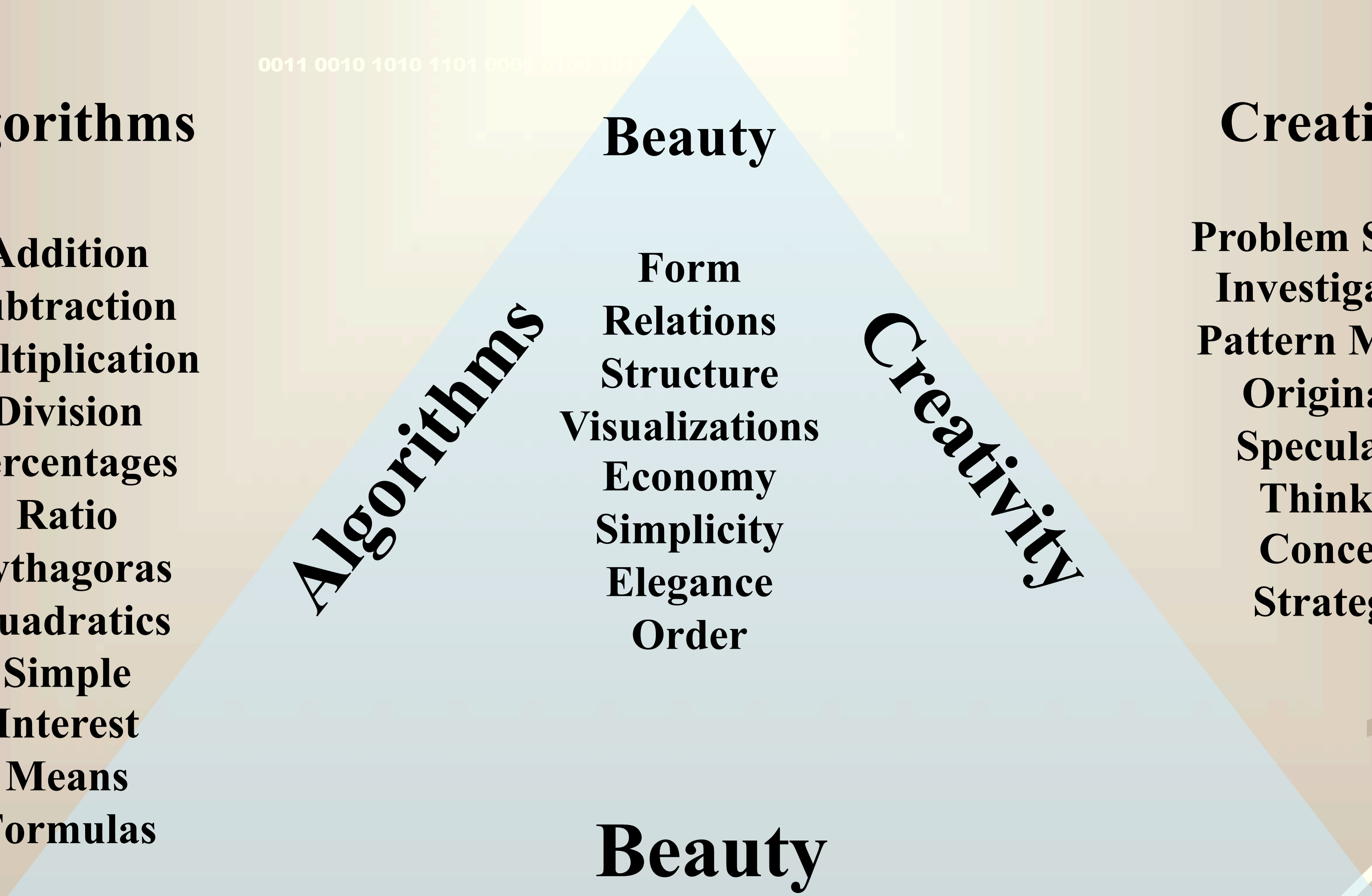
***Janitors Wanted, $42,000 starting salary, Ph.D. required***

***Central New York Computer Club: "Learning to Use Spell Check" Meat in the Lobby of Supper 8 Motel, 8-9 p.m., Wednesday***

# Whitcombe's Model of the Mathematical Mind

**Algorithms**

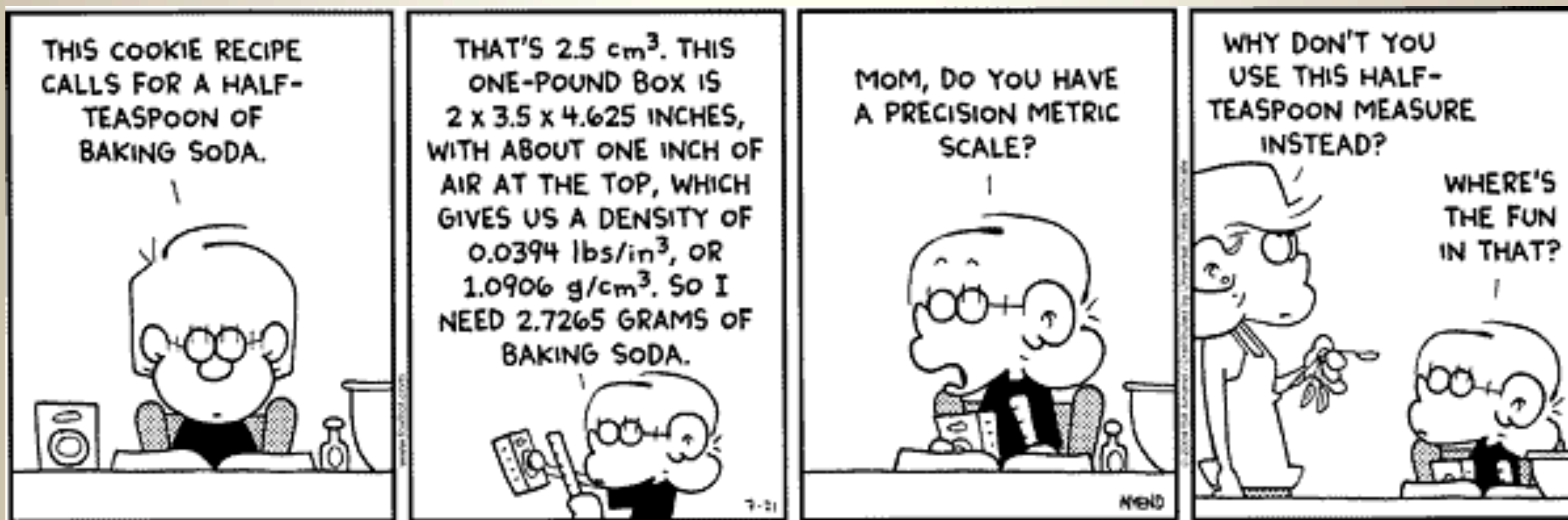
Addition
Subtraction
Multiplication
Division
Percentages
Ratio
Pythagoras
Quadratics
Simple
Interest
Means
Formulas




**Creativity**

Problem Solving
Investigations
Pattern Making
Originality
Speculation
Thinking
Concepts
Strategies

# The Fun in Math




## Can you read this Math Poem?

$$((12 + 144 + 20 + (3 * \sqrt{4})) \div 7) + (5 \times 11) = 9^2 + 0$$

Jon Saxton

$$((12 + 144 + 20 + (3 * \sqrt{4})) \div 7) + (5 \times 11) = 9^2 + 0$$

A dozen, a gross, and a score,

plus three times the square root of four,

divided by seven,

plus five times eleven,

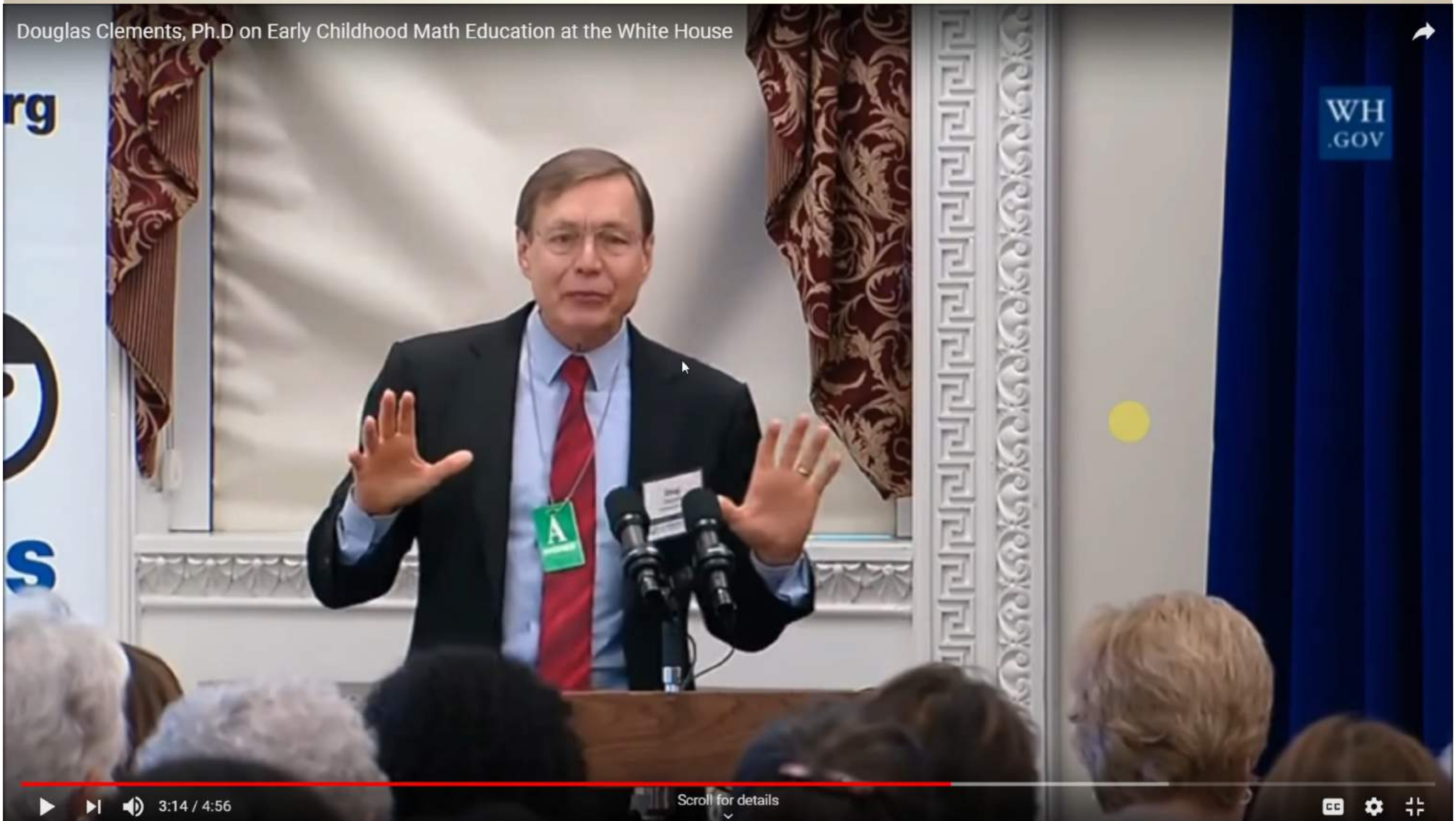
equals nine squared and not a bit more.

Found on-line at http://ctl.unbc.ca/CMS/poems.html

# Parting Thoughts

- “Any” fruit of human endeavor shows creativity, if you think about it. The interesting question to me is this: Why is it that a student who is only playing other people's music instinctively understands that those composers were creative, and that s/he might aspire to the same kind of creativity -- or, in English class, instinctively understands that those writers were creative, even when s/he is just reading their creations and answering quiz questions about them -- but doesn't have the same instinctive understanding that Euclid and Newton and Pascal and Gauss and Euler were creative mathematicians?
- The most obvious answer has to do with the way these disciplines are taught.

(Bogomolny, 2000)




https://youtu.be/WnkSl-obh3c