Title: Upscaling interpretation of nonlocal fields, gradients and divergences

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Abstract:
The interrelations between weight-function upscaling (measurement) and the definition of various nonlocal operators is explored. Let \( < f > = f \ast g \) where \( f \ast g \) is the convolution product which represents the effect of upscaling via an instrument (defined by g) on a field variable f and its localized counterpart. Nonlocal field variables are defined and employed for upscaling. In this talk it will be shown via Fourier transform, for judicious choice of the arbitrary function \( G_p \), that \( G_p f(x) = < \text{grad } f > (x) \), where \( G_p f (x) \) is the nonlocal gradient of f and \( \text{grad } f \) is the classical gradient. Upscaled representations for the adjoint of \( G_p \) and the nonlocal divergence are obtained. A nonlocal self-diffusion equation is upscaled and written in terms of nonlocal operators.

Keywords: nonlocal diffusion, nonlocal field variables, nonlocal gradient, nonlocal divergence and upscaling