Portfolio Project

Directions

1. **For each conjecture**, determine whether the conjecture is true or false. If the conjecture is true, state an appropriate theorem or proposition and prove it. If a conjecture is not true, provide a counterexample to show that it is false. In addition:
   - If a biconditional statement is found to be false, you should clearly determine if one of the conditional statements within it is true. In that case, you should state an appropriate theorem for this conditional statement and prove it.
   - If you determine that a conjecture to prove that that two sets are equal is false, then you should also determine if one of the sets is a subset of the other set. If so, you should state an appropriate theorem or proposition and prove it.
   - If you determine that a conjecture is to prove that a certain function is a bijection is false, then you should determine if the function is an injection or is a surjection. You should then state an appropriate theorem and prove it.
2. See the **Guidelines for the Portfolio Project** for the important due dates and other rules for the Portfolio Project.
3. **Honor System**. All work that you submit for the Portfolio Project must be your own work. This means that you may not discuss the portfolio project with anyone except the instructor of the course and may not use any resources other than the textbook.
4. **Electronic Submission of Portfolio Problems**. Each solution or proof must be done on a word processor capable of producing the appropriate mathematical symbols and equations. Microsoft Word and its Equation Editor, which is available on the student network, is one such word processor.

   Each solution or proof for a portfolio problem must be submitted to the instructor electronically through the Digital Drop Box that is on the course web page (through Grand Valley's Blackboard system). The instructor will make comments on the problem and return them to the student using the Digital Drop Box.

Portfolio Problem #1

The **Pythagorean Theorem**for right triangles states that if a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$. For example, if the lengths of the legs of a right triangle are 4 and 7 units, then $c^2 = 4^2 + 7^2 = 63$, and the length of the hypotenuse must be $\sqrt{13}$ units (since the length must be a positive real number).

Prove that if m is a real number the lengths of the three sides of a right triangle are m, $m + 7$ and $m + 8$ units, then the length of the hypotenuse must be 13 units.

Portfolio Problem #2

Does the graph of the equation $y = 3x^2 + 5x + 4$ have any points in which both coordinates are odd integers? Justify your conclusion.

One way to approach this is to develop and prove a theorem about $3n^2 + 5n + 4$ when n is an odd integer.

Portfolio Problem #3

Conjecture.
For each integer a, if there exists an integer n such that a divides $(8n + 7)$ and a divides $(4n + 1)$, then a divides 5.

Portfolio Problem #4

Conjecture.
For all integers a and b, if $ab \equiv 7 \pmod{12}$, then either $a \equiv 1 \pmod{12}$ and $b \equiv 7 \pmod{12}$ or $a \equiv 7 \pmod{12}$ and $b \equiv 1 \pmod{12}$.

Portfolio Problem #5

Conjecture.
For all nonzero integers a and b, if $a + b \neq 3$ and $9a + b \neq 1$, then the equation $ax^3 + bx - 3 = 0$ does not have a solution that is a natural number.

Portfolio Problem #6

Conjecture.
Prove that for each integer a, if $a \not\equiv 0 \pmod{5}$, then $a^2 \equiv 1 \pmod{5}$ or $a^2 \equiv 4 \pmod{5}$. Then prove that for each integer a, if 5 divides a^2, then 5 divides a. Finally, prove that $\sqrt{5}$ is an irrational number.

Portfolio Problem #7

Conjecture.
For all sets A, B, and C that are subsets of some universal set U, $(A - B) - C = A - (B - C)$.

Portfolio Problem #8

Two prime numbers that differ by 2 are called **twin primes**. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Determine at least two other pairs of twin primes.

Conjecture.
For all natural numbers p and q if p and q are twin primes other than 3 and 5, then $pq + 1$ is a perfect square and 36 divides $pq + 1$.

Portfolio Problem #9

Let $y = \ln x$. Determine $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$, $\dfrac{d^3y}{dx^3}$, and $\dfrac{d^4y}{dx^4}$.

Let n be a natural number and let $y = \ln x$. Formulate a conjecture for a formula for $\dfrac{d^ny}{dx^n}$. Then use mathematical induction to prove your conjecture.

Portfolio Problem #10

Conjecture.
For each natural number n, $\left(\dfrac{n^5}{5} + \dfrac{n^4}{2} + \dfrac{n^3}{3} - \dfrac{n}{30}\right)$ is an integer.
