SUMMR 2017

The Summer Undergraduate Michigan Mathematics Research Conference

July 14, 2017

Time	**Session**	**Location**
10:00 – 10:30 am	Check-in and Light Breakfast	KHS 2225
10:30 – 10:50 am	Session 1	KHS 2207, 2213, 2219
10:55 – 11:15 am	Session 2	KHS 2207, 2213, 2219
11:20 – 11:40 am	Session 3	KHS 2207, 2213, 2219
11:40 – 12:55 pm	Lunch	Fresh Food Company
1:00 – 1:50 pm	Plenary	KHS 1101
2:00 – 2:20 pm	Session 4	KHS 2207, 2213, 2219
2:25 – 2:45 pm	Session 5	KHS 2207, 2213, 2219
2:50 – 3:10 pm	Session 6	KHS 2207, 2213, 2219

All sessions are in the Kindschi Hall of Science (KHS), except for lunch. The back of your lunch ticket includes a map with directions to the Fresh Food Company.

Session 1: 10:30 – 10:50 am

An Application of Galois Theory to Explore Properties of Recursive Polynomials Room #2207
Mohit Bansil - MSU

In this talk we use Galois theory to understand the nature of the roots of recursively defined polynomials. Two cases where the exact roots are understood are the Fibonacci polynomials, $F_n(x)$ defined by $F_1(x) = 1, F_2(x) = x, F_n(x) = xF_{n-1}(x) + F_{n-2}(x)$, and the Lucas polynomials, $L_n(x)$ defined by $L_1(x) = x, L_2(x) = x^2 + 2, L_n(x) = xL_{n-1}(x) + L_{n-2}(x)$. In these cases we show that the Galois groups are abelian and hence solvable. We extend this result to an infinite family of recursively defined polynomials, namely $H_0(x) = a(x), H_1(x) = a(x)p(x), H_n(x) = p(x)H_{n-1}(x) + q(x)H_{n-2}(x)$, where a, p, q are integer polynomials. We will show the Galois group of H_n is solvable under mild conditions. In addition we present results about the inexpressibility of the roots of other recursively defined polynomials by examining the solvability of their Galois groups. For example, via computation with Magma (a software package for rigorous computations in algebra), we can show if $G_1(x) = -1, G_2(x) = x - 1, G_n(x) = xG_{n-1}(x) + G_{n-2}(x)$ then the Galois group of G_n is the n-th symmetric group, $1 \leq n \leq 15$.

Curves of Constant Curvature and Torsion in the 3-Sphere Room #2213
Rahul Sahay - CMU
Jared Williams - CMU

It is well-known that the local geometry of a smooth curve in Euclidean space $\mathbb{R}^3$ is given by the curvature and torsion functions κ and τ defined along the curve. When these functions are non-zero constants, we obtain a *helix*, which is the trajectory of a point whose motion is the superposition of a circular motion in a plane, and a rectilinear motion in a direction perpendicular to this plane. Helices are interesting in geometry since they are invariant under a one-parameter group of isometries, and arise frequently in applications, for example in biology.

In this presentation, we study curves of constant curvature and torsion in the three-sphere $\mathbb{S}^3$, where the (geodesic) curvature and torsion are defined with respect to the round metric. Locally, such curves (which we call *helices* in $\mathbb{S}^3$) are not very different from Euclidean helices, but thanks to the fact that $\mathbb{S}^3$ is compact and has positive curvature, one expects significant departures in the global behavior.

We show by directly solving the covariant Frenet-Serret equations on the round sphere $\mathbb{S}^3$, that a helix in $\mathbb{S}^3$ lies on a *Clifford torus*, i.e., a 2-dimensional submanifold of $\mathbb{S}^3$ isometric to the product of two circles. The helix may be thought of the trajectory of a point whose motion is obtained by superimposing two uniform circular motions at angular frequencies ω_1 and ω_2 along these two circles. The two angular frequencies satisfy the condition

$$\omega_2 > 1 > \omega_1,$$

a restriction which arises from the geometric constraint of accommodating a curve of nonvanishing curvature and torsion on a Riemannian manifold of positive sectional curvature. We show further that (just like in the Euclidean case) two helices with the same curvature and torsion are congruent, i.e., there is an isometry of $\mathbb{S}^3$ which carries one to the other.

The global behavior of the helix is shown to be determined by the ratio $\frac{\omega_2}{\omega_1}$ of the two angular frequencies. If this ratio is a rational number, the helix is periodic, and its image is a compact embedded submanifold of $\mathbb{S}^3$. On the other hand, when this ratio is irrational, the helix is dense in the Clifford torus on which it lies. These behaviors are very different from that of a Euclidean helix, which is always noncompact and an embedded submanifold of $\mathbb{R}^3$.

Geometries from Groups Room #2219
Casey Koch-LaRue - GVSU

In this talk we will present a new technique for geometrically representing the subgroup structure of a finite group. The approach is to create a finite geometry from the set of subgroups. Using word lengths, distances between the elements of a group can be measured. We then measure distances in the set of subgroups of a finite group using the Hausdorff distances between subgroups. Finite geometries emerge in which points are the subgroups of a finite group and lines are sets of subgroups of the same finite group. We will present several examples of geometries that correspond to finite cyclic groups and also show how the characteristics of families of these geometries are connected with the prime factorizations of the orders of the related groups. This research was conducted as part of the 2017 Student Summer Scholars Program at Grand Valley State University.

Session 2: 10:55 – 11:15 am

Anti-Games on Steiner Triple Systems Room #2207

Sophie Mancini - James Madison University
Jacob Van Hook - Penn State University

The card game SET can be turned into a two-player tic-tac-toe style game: All cards are laid out on a table, and two players alternate taking one card at a time. The winner is the first to hold a set in their hand. "Anti-SET" is a variation of this game in which the first player to hold a set loses the game. Previous researchers found a first-player winning strategy for Anti-SET and related games. In this talk, we generalize the game of Anti-SET to a larger category of combinatorial objects called Projective Steiner Triple Systems (STSs). These well-studied objects share many of the key geometric features that determine the winning strategy for Anti-SET. We demonstrate a winning strategy for these games under certain assumptions using geometric substructures of STSs. This research was conducted as part of the 2017 REU program at Grand Valley State University.

Name That Bird! Using Neural Networks to Identify Birds Room #2213

Russell Houpt - Hope College

Can a computer learn to identify a bird by analyzing samples of its song? This research explores how neural networks can be used to identify different birds from recordings of their songs. In this talk, we will explore what neural networks are, how they work, and what techniques were employed to teach the programs how to quickly and accurately identify birds. In earlier work, a research group at Hope College made progress on this question by using neural networks to classify bird songs on a somewhat limited scale. Our results extend this work by using similar techniques on larger data sets, improving the accuracy and speed of the analysis, and modifying the existing algorithms to take advantage of multiple core computers.

Improving Automated Methods for Cell Identification in Calcium Images Room #2219

Meraiah Martinez - Benedictine College
Hanyu Gao - MSU

New optical imaging technologies have the potential to revolutionize neuroscience, but are hindered by inaccurate automated cell identification. We are comparing results from current automated cell-sorting methods to careful human judgement. We are investigating the current algorithms which use singular value decomposition and non-negative matrix factorization. We are calculating single pixel statistics which are aiding our efforts to improve the automated methods in identifying neurons. We are exploring the correlations within the background noise of multiple neurons to augment our ability to identify, and then remove, the unnecessary background noise.

Session 3: 11:20 – 11:40 am

What Bird Was That? Feature Extraction of Recorded Bird Songs for Neural Networks Room #2207
Sarah Seckler - Hope College

In the past, researchers at Hope have worked towards identifying birds from recorded bird songs through using wavelets, image processing and neural networks. The general aim of our project is to extend this work to provide greater computational efficiency and accuracy in identification of bird songs. In this talk I will focus on taking a recorded bird song signal and extracting data from it to make it a suitable input for a neural network. This feature extraction process will involve using wavelets and related methods to create an image called a scalogram, encoding the key aspects of the sound including frequency and time. Our work focuses primarily on finding more efficient ways to extract these images, allowing us to analyze much larger data sets.

On the Properties of kth-Order Fibonacci-like Polynomials Room #2213
Katherine Arneson - St. Olaf College
Jason Bruce - University of Rochester
Alexandra Embry - Indiana University

For a fixed positive integer $k \geq 1$, we introduce recursively defined polynomial sequences given by $G_n^{(k)}(x) = xG_{n-1}^{(k)}(x) + G_{n-k}^{(k)}(x)$ and $H_n^{(k)}(x) = xH_{n-1}^{(k)}(x) - H_{n-k}^{(k)}(x)$ with initial conditions $G_0^{(k)} = G_1^{(k)} = \ldots = G_{k-1}^{(k)} = 1$ and $H_0^{(k)} = H_1^{(k)} = \ldots = H_{k-1}^{(k)} = 1$. Note that for $k = 2$, $G_n^{(2)}(x) = F_{n-1}(x) + F_{n-2}(x)$ where $F_n(x)$ are the Fibonacci polynomials, with $F_0 = 1$ and $F_1 = x$. Further, when $x = 1$, $G_n(1) = F_n(1)$. In this talk we present some interesting properties of $G_n^{(k)}$ and $H_n^{(k)}$. In particular, we will establish, for all $k \geq 3$, a uniform asymptotic result for the maximal real roots of $H_n^{(k)}$, provide similar results for the minimal real roots of $G_n^{(k)}$, and explore interesting integer sequences that arise from the polynomials.

Optimal Packings of Four Equal Circles on a Flat Klein Bottle Room #2219
Sean Haight - Western Washington University
Quinn Minnich - Millersville University

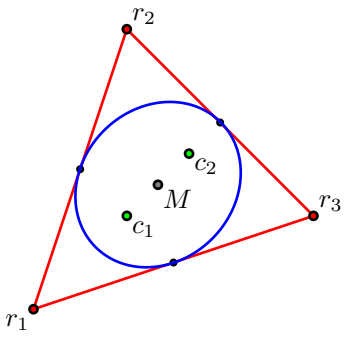
The study of maximally dense packings of disjoint equal circles is a problem in Discrete Geometry. The optimal densities and arrangements are known for packings of small numbers of equal circles into hard boundary containers, including squares, equilateral triangles and circles. In this presentation, we will explore packings of small numbers of equal circles onto a boundaryless container called a flat Klein bottle. Using numerous figures we will introduce all the basic concepts (including the notion of a flat Klein bottle, an optimal packing and the graph of a packing), illustrate some maximally dense arrangements, and outline the proofs of their optimality. This research was conducted as part of the 2017 REU program at Grand Valley State University.

Plenary: 1:00 – 1:50 pm

The Geometry of Polynomials Room #1101

Matt Boelkins - GVSU

In the *geometry of polynomials*, we seek to understand relationships among certain sets connected to polynomial functions. For instance, given a member of a family of polynomials, we may be interested in how the set of critical numbers is related to the corresponding set of the polynomial's zeros, with the goal of making general observations that apply to the entire family regarding the relationship between these sets.



In this talk, we'll discuss several fundamental historical results from the geometry of polynomials, including the Gauss-Lucas Theorem and Marden's Theorem. We will also survey some recent developments centered on the idea of *polynomial root-dragging*, the study of how continuously changing one or more roots of a polynomial function affects various properties of the function. Along the way, we'll consider a few results proved by undergraduate students and witness beautiful interplay between Euclidean geometry and calculus in the context of complex cubic polynomials.

Session 4: 2:00 – 2:20 pm

Model choice and future prediction accuracy in time series for disease incidence Room #2207

Reagan Spindler - Hope College

One of the goals of time series models for disease incidence data is the prediction of future disease counts. Many such models exist, some of which utilize information from covariates. We introduce a Bayesian neural network time series model for predicting dengue fever incidence in Singapore, which utilizes Singaporean precipitation data. A comparison is made between this neural network model and a time series model which does not use any covariate information. A method for choosing between the models which optimizes future prediction accuracy is suggested as well.

Linear Fractional Transformations of the Polydisc $\mathbb{D}^2$ Room #2213

Amanda L. Cowell - University of Michigan-Dearborn
Timothy A. Hollman - University of Michigan-Dearborn
Sarah L. Strikwerda - Calvin College
Erin E. Wallace - Texas A&M University

In this talk, we address linear fractional transformations, commonly referred to as Möbius transformations, in $\mathbb{C}^2$ as they act on the polydisc $\mathbb{D}^2$. Previously, linear fractional transformations (LFT's) have been studied on the unit disc in $\mathbb{C}$ and on the unit ball in $\mathbb{C}^n$. Through investigating this rich family of holomorphic maps we determine the image that is formed on the polydisc. We explore the amount of fixed points LFT's have and their use in constructing different LFT's. Finally, modeling Cowen's work on the unit ball in $\mathbb{C}^n$, we solve Schröder's equation using the matrix representation of LFT's.

Mathematical Analysis of the Effects of Age, Education, and APOE-4 Gene on Human Cognitive Function

Room #2219

Emily Dorn - Olivet College
Philip Grossweiler - University of Texas Rio Grande Valley
Savannah Swiatlowski - Central Michigan University

Extensive research has been conducted on Alzheimer's Disease and other forms of Dementia, yet much remains to be understood. A common quantitative measure of cognitive impairment in patients with Dementia is their score on the thirty-point questionnaire called the Mini-Mental State Examination (MMSE). For the initial study, we will investigate the relationship between MMSE scores and three factors known to have an effect on the rate of cognitive decline: age, years of formal education, and the APOE-4 gene. Other factors may be incorporated into the model in a future study. Using MATLAB and data from the National Alzheimer's Coordinating Center (NACC), our main goal is to develop an age-dependent differential equation model that will incorporate these factors:

$$\frac{\partial M}{\partial a} + k_1 \frac{\partial M}{\partial e} = k_2$$

where a denotes age, e denotes education, M denotes MMSE score, and k_1 and k_2 are constants. We hope to use this differential equation model to improve the understanding of how these factors work collectively to affect the degeneration of cognitive function in patients with Alzheimer's Disease and other Dementia disorders.

Session 5: 2:25 – 2:45 pm

Generalized Catalan Numbers Room #2207

Emily Dautenhahn - University of Kentucky
Hannah Pieper - Oberlin College

There are numerous sets of combinatorial objects that are counted by the Catalan numbers $C(n) = \frac{1}{n+1}\binom{2n}{n}$, and many mathematicians have constructed bijections between these sets. The Catalan numbers can be generalized using parameters p and r, which correspond to the Raney numbers $R_{p,r}(n) = \frac{r}{np+r}\binom{np+r}{n}$, where $R_{2,1}(n) = C(n)$. We present the results of our research on sets counted by generalized Catalan numbers, including combinatorial bijections between the sets to demonstrate that they have the same cardinality. This research was conducted as part of the 2017 REU program at Grand Valley State University.

Spectrum of $\Box_b^t$ on the 3-sphere Room #2213

Ravikumar Ramasami - University of Michigan-Dearborn
Dylan Byers - University of Toledo
Madelyne Brown - Bucknell University
Tawfik Abbas - Michigan State University

When is a CR-manifold CR-embeddable into $\mathbb{C}^n$? The Whitney embedding theorem states that any manifold is embeddable into Euclidean space at most twice its dimension, but this may not preserve the CR-structure. The question was partially answered by Boutet de Monvel in 1975, who stated that if the CR-manifold has real dimension at least 5, then it can always be CR-embedded into $\mathbb{C}^N$ for some N. It turns out that if a CR-manifold has real dimension 3, this fails. Rossi showed in 1965 that the CR-manifold $(\mathbb{S}^3, \mathcal{L}_t)$ is not CR-embeddable, where $\mathcal{L}_t$ is a slight perturbation of the normal CR-structure $\mathcal{L}$ on $\mathbb{S}^3$. Later in 1986, Kohn showed that CR-embeddablity is equivalent to showing that the tangential Cauchy-Riemann operator $\overline{\partial}_{b,t}$ does not have closed range. We tackle the same problem of embeddability, but from the perspective of spectral analysis. In particular, we show that if the Kohn Laplacian $\Box_b^t$ contains 0 in its essential spectrum, then $(\mathbb{S}, \mathcal{L}_t)$ is not CR-embeddable. To get the spectrum of $\Box_b^t$, we found spherical harmonic spaces such that the matrix representation of $\Box_b^t$ is tridiagonal on these finite-dimensional subspaces. We then exploited this fact to bound the smallest eigenvalue of $\Box_b^t$, and find a sequence of eigenvalues that converge to 0.

Droning on about Sand Dunes and Machine Learning Room #2219

Keri Haddrill - Hope College

Gusty winds expand across the dunes and send sand flying everywhere. They brush past the vegetation grasping at the sand, and together they alter the formation of dunes. The Saugatuck Harbor Natural Area (SHNA) along Lake Michigan has numerous open sand dunes at risk for extinction. Thus, our project's goal is to map the changes of vegetation over time. The first step in mapping SHNA is comparing the surface reflectance images collected by a drone and field biomass measurements from a few small selected areas. This information allows us to create a convolutional neural network that can approximate above-ground biomass and therefore produce a biomass map for the entirety of SHNA.

Session 6: 2:50 – 3:10 pm

Dynamics of the family $\lambda(z+\frac{1}{z}+1)$ Room #2207
Anzhané Lance - FSU

We study the dynamics (i.e. behavior under repeated iteration) of the family of quadratic rational maps $\lambda(z+\frac{1}{z}+1)$, where $\lambda \in \mathbb{C}$. In this talk, we will focus on describing the topology of Julia sets for parameters λ where this family has an attracting fixed point and/or attracting period 2 cycle, and if time permits, we will discuss more properties of this family.

Predicting Separability from Partial Preferences in Referendum Elections Room #2213
Tasha Fu - UMBC
David Shane - GVSU

In referendum elections, voters are often required to cast simultaneous votes on multiple questions or proposals. The separability problem occurs when a voter's preferences on the outcome of one or more proposals depend on the predicted outcomes of other proposals. Preferences that are free from such interdependencies are said to be separable. Determining the extent to which a voter's preferences are separable often requires extensive information about their ranking of all of the possible election outcomes. In this talk, we will explore what conclusions about separability can be made from partial voter preferences. Our work has potential applications to election sequencing, particularly when only incomplete information about voter preferences is available. This research was completed as part of the Summer Mathematics REU at Grand Valley State University.

Stochastic Modeling of Chemotherapy Treatment for Cancer with a Large Deviation Principle Room #2219
Zachary Bezemek - MSU
Robert Lucas - University of Michigan - Dearborn
Casey Saenz - University of Michigan - Dearborn
Zachary Shay - NYU

We develop and investigate an equivalent stochastic differential equation (ESDE) model for an existing ordinary differential equation (ODE) model for tumor-immune system interactions with chemotherapy, with particular interest in how the stochastic component affects the original ODE system. We calculate the probability of the rare event of tumor self-eradication after premature termination of adjuvant chemotherapy treatment by establishing the existence of a large deviation principle (LDP) in a simplified version of the ESDE model, a phenomenon that is nonexistent in the ODE system. Lastly, we calculate the expected times for eradication of the tumor cells and for the patients' immune system health to meet critical levels, establishing formulas for these results that agree with our numerical simulations.
